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Authors:
Gabriel Chodorow-Reich
Plamen T. Nenov
Vitor Santos
Alp Simsek

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# Stock Market Wealth and Entrepreneurship*

Gabriel Chodorow-Reich[†] Plamen T. Nenov[‡] Vitor Santos[§] Alp Simsek[¶]

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**Abstract**

We use data on stock portfolios of Norwegian households to show that stock market wealth increases entrepreneurship by relaxing financial constraints. Our research design isolates idiosyncratic variation in household-level stock market returns. An increase in stock market wealth increases the propensity to start a firm, with the response concentrated in households with moderate levels of financial wealth, for whom a 20 percent increase in wealth due to a positive stock return increases the likelihood to start a firm by about 20%, and in years when the aggregate stock market return in Norway is high. We develop a method to study the effect of wealth on firm outcomes that corrects for the bias introduced by selection into entrepreneurship. Higher wealth causally increases firm profitability, an indication that it relaxes would-be entrepreneurs' financial constraints. Consistent with this interpretation, the pass-through from stock wealth into equity in the new firm is one-for-one.

**JEL Classification: E22, E44, L26, G50**

**Keywords: entrepreneurship, financial frictions, selection correction**

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[†]Harvard University and NBER. Email: chodorowreich@fas.harvard.edu

[‡]Norges Bank and Norwegian Business School (BI). Email: plamen.nenov@norges-bank.no

[§]Norwegian Business School (BI). Email: vitor.santos@bi.no

[¶]Yale School of Management, NBER, and CEPR. Email: alp.simsek@yale.edu

# 1 Introduction

New business creation constitutes an integral part of economic growth and provides an important pathway to individual wealth accumulation (Buera et al., 2015; Smith et al., 2019; Bhandari et al., 2022). Does initial wealth affect who starts a firm or the type of business they build? In theory, higher wealth might increase business entry and profits by allowing would-be entrepreneurs to overcome fixed costs of set up. Yet, establishing a causal effect of wealth on entry faces the challenge that high wealth individuals may have other traits that make them more likely to start businesses (Hurst and Lusardi, 2004). Even granting a causal effect, determining why wealth matters or the effect of wealth on business outcomes such as profitability faces the further difficulty that an entry effect causes non-random selection on talent across entrepreneurs with different initial wealth (Buera, 2009).

We make four contributions. First, we study the effect of *stock market wealth*, the most volatile component of household financial wealth but one that has received comparatively little attention in the literature on entrepreneurship. Second, we show a causal effect of wealth on entry using a research design that compares individuals with the same *ex ante* stock portfolio characteristics but different *ex post* market returns arising from idiosyncratic holdings. Third, we isolate the causal effect of wealth on business outcomes by demonstrating a rank preservation property in a simple but general model of entrepreneurial choice. Fourth, in our data higher initial wealth causally increases key business variables, including firm profits and owners' equity, providing evidence that wealth matters to entrepreneurship by relaxing financial constraints.

To frame the empirical analysis, we start by describing the model and testable implications. Individuals differ in initial wealth and potential business productivity. Since our empirical approach isolates random variation in wealth, we assume that initial wealth is conditionally independent of productivity. An individual starts a business only if her utility as an entrepreneur exceeds her utility from wage employment. Greater wealth can affect this trade-off either by allowing for higher business profits (e.g. because of financial constraints) or by increasing the utility from running a business (e.g. because entrepreneurship provides non-pecuniary benefits). Comparing individuals with high and low initial wealth correctly identifies the magnitude of this entry effect (see Proposition 1) but does not distinguish the reason for it. The causal effect of initial wealth on firm profits provides a key moment to distinguish financial constraints from a non-pecuniary benefits explanation of the wealth effect on entry, as only under the former does higher wealth relax operating constraints and result in higher profits.

The effect of wealth on business outcomes such as profits is however harder to identify because we observe firm outcomes *only for entrants*, and entrants with different initial wealth have non-random differences in productivity. While this insight follows directly from a standard model of entrepreneurial selection, simple comparisons of outcomes of high and low wealth entrepreneurs remain prevalent in empirical work. We show that, under a natural monotonicity assumption, the model satisfies a rank preservation property that enables us to match entrants by productivity and estimate the causal effect of wealth on firm-level outcomes (see Proposition 2). Intuitively, while we do not directly observe productivity, we observe firm size, which is a proxy for the firm's productivity rank when size increases with productivity. Using this proxy, we remove from the comparison the left tail of the productivity distribution of the firms with high initial wealth, as these firms would not have existed if the founder instead had low initial wealth. This correction for non-random entry enables us to identify the causal effect of founder wealth on business outcomes. Propensity score reweighting extends the result to the realistic case of independence between wealth and ability *conditional on observables* (see Proposition 3).

We bring the model's insights to the data using administrative records from Norway for both households and firms. We merge several administrative data sets, including a registry of security-level holdings of Norwegian stocks, total household financial wealth from tax records, labor market history from the employer-employee register, and firm balance sheet and income statements. In short, we observe household financial wealth, portfolio allocation at the individual stock level, business ownership, and firm-level outcomes. We define an entrepreneur as an individual who owns at least 1/3 of the book value of stocks in an incorporated non-financial firm, and where the firm has at most 3 stock owners. To remove shell companies we further require that the firm has at least one employee in the year of foundation or subsequent year and either holds no public equities or employs a worker who is not a member of the entrepreneur's household.

Our first main result compares entry into entrepreneurship across individuals with different stock market returns. There are two important threats to causal identification. First, realized stock market returns may correlate with other factors that affect the entrepreneurship decision. For example, home bias in portfolio choice could result in better returns in periods when the individual's industry or local area is booming. We address this concern by including sector$\times$time and geography$\times$time fixed effects. Second, individuals more likely to become entrepreneurs might hold systematically different portfolios, for example if risk-tolerant individuals both hold riskier portfolios and are more likely to start firms. Following the results in Borusyak and Hull (2021), we address this concern by including flexible controls for *ex ante* portfolio characteristics such as the market beta. Our research design therefore isolates variation in market returns that comes from random realizations of the idiosyncratic component of portfolio holdings across portfolios with the same *ex ante* characteristics. In our setting, more than half of households with directly held stocks have only one or two holdings, generating substantial idiosyncratic return variation.

We find strong evidence that greater stock market wealth increases entrepreneurship. Pooling across all observations, a 20 percentage point increase in total financial wealth due to an idiosyncratic stock return raises entry into entrepreneurship by about 1/10th of the sample average rate. This overall effect masks important underlying heterogeneity: individuals with sufficiently high initial wealth exhibit essentially no effect of higher wealth on entrepreneurship, while for lower wealth individuals the marginal effect is essentially doubled, with a 20 percentage point increase in wealth raising the entry rate by about 20%. We also find that the positive marginal effect comes only from positive returns or in years when the overall stock market does well. We demonstrate robustness along several dimensions, including controlling for initial wealth and labor income or for the sector of the individual's largest stock holding. The positive effect of wealth on entry also holds when restricting the sample to similarly under-diversified households, those that hold less than 3 stocks and whose main stock holding is among the 20 most popular companies among small investors traded on the Norwegian stock exchange. Placebo exercises demonstrate no response of entrepreneurship to future stock returns.

Our second set of results apply the model's selection correction to obtain the causal effect of wealth on firm outcomes. We find sizable positive effects of higher portfolio returns on firm balance sheet and income statement outcomes, including capital, sales, employment, value added, and profitability. As shown in our model, the increase in profits indicates a role for financial constraints. Two additional results further this interpretation: (i) a marginal increase in stock wealth results in a nearly one-to-one increase in owners' equity in the new firm, and (ii) entrepreneurs with higher returns finance their larger businesses by actively liquidating stocks.

Together, our results provide evidence that higher stock market wealth increases entrepreneurship and results in more profitable firms at creation. This evidence complements research using other sources of windfall gains such as housing capital gains or lottery winnings. Understanding the effects of stock market fluctuations in particular are important because the stock market accounts for a large share of total wealth fluctuations. Furthermore, the stock market setting is special in that it allows for examination of both negative and positive wealth changes and because stock wealth rises when the overall stock market does well, which our evidence shows is also exactly when such wealth matters most to entrepreneurship. Finally, the relaxation of financial constraints provides a key transmission channel for these results.

**Related literature.** Our paper makes a direct contribution to the literature on wealth and entrepreneurship and in particular to the academic debate on the importance of liquidity constraints for business creation.[1] In two early seminal contributions, Evans and Jovanovic (1989) and Evans and Leighton (1989) find a positive association between individual wealth and the propensity to start a business using data from the National Longitudinal Survey of Youth. Using a structural approach, Evans and Jovanovic (1989) argue that this relationship is likely causal. Similar correlational findings have been reported in Blanchflower and Oswald (1998), Fairlie (1999), Quadrini (1999), and Gentry and Hubbard (2004) among others.

On the other hand, Hurst and Lusardi (2004) find a flat relationship in the Panel Study of Income Dynamics between wealth and entry into entrepreneurship for most of the wealth distribution and a strong positive relationship only at the top. In addition, they find no evidence that individual wealth matters more for entry in high starting-capital industries. More recently, Bhandari et al. (2022) use administrative data from the IRS and Social Security Administration and find that entrants into self-employment have *lower* asset incomes prior to entry.

The lack of consensus on the relationship between wealth and entrepreneurship spurred a literature that looks for exogenous shocks to wealth. Several studies have found that individuals receiving an inheritance are more likely to become entrepreneurs (Holtz-Eakin et al. (1994), Lindh and Ohlsson (1996), Blanchflower and Oswald (1998), Andersen and Nielsen (2012), Fairlie and Krashinsky (2012)). Hurst and Lusardi (2004) challenge this approach by showing that both past and *future* inheritances predict entry into entrepreneurship, suggesting that inheritance may correlate with other factors such as risk tolerance or preferences that determine entry.

Another form of variation in wealth comes from lottery winnings or other cash windfalls. Lindh and Ohlsson (1996) report a positive effect of lottery winnings on firm creation in Sweden while Cesarini et al. (2017) find that lottery winnings reduce self-employment income in Swedish data. Cespedes et al. (2021) investigate the effect of a retail business receiving a bonus payment for selling a winning jackpot ticket and find both an intensive and extensive (serial entrepreneurship) effect that depends on the size of the lottery windfall. Using U.S. administrative tax records, Golosov et al. (2021) find a positive effect of lottery winnings on the propensity to report small ($< \$15,000$) self-employment income but no effect on

[1] Parker (2018) provides a comprehensive treatment of the economics of entrepreneurship including the link between wealth and entrepreneurship. See also Kerr and Nanda (2011). In addition to empirical studies, the idea of financial constraints impacting entry into entrepreneurship has also been explored theoretically and quantitatively in Banerjee and Newman (1993), Aghion and Bolton (1997), Piketty (1997), Cagetti and De Nardi (2006), Buera and Shin (2013), Moll (2014), and Morazzoni (2023), among others. See Buera et al. (2015) for a review of that literature.

transitioning to more substantial business activity. Bermejo et al. (2022) find a positive effect of winning the Spanish Christmas lottery on regional firm creation, which they argue is mediated via a credit constraint channel. Finally, Bellon et al. (2021) find a positive effect on incorporated business creation from cash windfalls due to shale oil exploration contracts in Texas.

The third wealth shock used in the literature is to housing wealth (Hurst and Lusardi (2004), Adelino et al. (2015), Corradin and Popov (2015), Schmalz et al. (2017), Jensen et al. (2022), Kerr et al. (2022)). With the prominent exception of Hurst and Lusardi (2004), most of this literature finds that housing wealth increases business creation. However, since variation in housing wealth is mostly regional there are inherent difficulties in establishing whether these effects are driven by local economic shocks, by higher wealth, or by higher collateral values. Schmalz et al. (2017) cleverly circumvent this difficulty by comparing local homeowners to renters and find that local homeowners are more likely to start a business compared to renters after a local house price appreciation. They also compare local homeowners with and without a mortgage on their house and show that the effects are present only for homeowners without a mortgage – a sign of higher housing wealth relaxing liquidity constraints, since in France homeowners with a mortgage cannot take on more mortgage debt.[2]

Relative to this large existing literature, and to the best of our knowledge, we are the first to propose and implement an empirical design featuring the impact of stock market wealth on business creation.[3] Among other differences, unlike lottery winnings our data contain both increases and decreases in wealth and thus point to potential non-linear effects of changes in wealth on entrepreneurship. Furthermore, our administrative data contain the near-universe of households and firms over a long time period with no top-coding, non-observability of assets, or self-reporting errors that have been an issue for much of the existing literature. Our main findings point to a robust causal and economically significant effect of stock market wealth on business creation.

Several earlier papers also examine the effect of wealth on firm-level outcomes. Using data on start-ups from Norway, Hvide and Møen (2010) find a negative relationship between wealth and start-up profitability at the top of the wealth distribution. Similarly, Andersen and Nielsen (2012) find that on average the firms created with unexpected inheritances have lower survival and profitability. Much of the rest of the literature reaches the opposite conclusion. Holtz-Eakin et al. (1994) show that firms whose owners receive inheritances tend

[2] In addition to collateral shocks some papers have considered the effects of credit market shocks on entrepreneurship (Black and Strahan (2002), Kerr and Nanda (2009), Fracassi et al. (2013), among others).

[3] In contemporaneous and complementary work, Chetty et al. (in progress) show that early employees at firms undergoing an IPO have higher subsequent entrepreneurship rates. Our work differs in focusing on more "ordinary" stock market participants who have not necessarily already joined newly-formed firms.

to survive longer. Schmalz et al. (2017) find that firms created by homeowners in periods of rising house prices are significantly larger at the time of creation than those started by renters and that such firms tend to survive longer and are therefore not "riskier" in the sense of having a higher probability of failure. Jensen et al. (2022) also find higher survival rates among firms created with higher housing wealth. McKenzie (2017) uses the random allocation of grants to business start-ups in Nigeria to show that both potential and existing entrepreneurs that receive a grant are both more likely to operate a business three years after the grant allocation and more likely to operate firms with ten or more workers.[4] Bermejo et al. (2022) find a positive and significant effect on firm size and survival using the Spanish Christmas lottery.

None of these papers attempts to separately identify the causal effect of wealth conditional on becoming an entrepreneur from the changing distribution of entrants induced by higher wealth.[5] Indeed, our selection model can help to reconcile the disparate findings across papers, since it implies that unconditional differences between firms started by high and low wealth entrepreneurs have theoretically ambiguous sign and could vary across institutional settings. Our selection correction echoes the structural approach in Evans and Jovanovic (1989), but provides a more direct mapping from data to results. After applying our selection correction, we find strong evidence that higher wealth causes better firm outcomes.

The literature has also debated whether to interpret a causal relationship between wealth changes and entrepreneurship as revealing liquidity constraints. For example, higher wealth could make individuals more risk tolerant or overly optimistic and, hence, also more likely to accept the non-diversification risk of starting a business (Kihlstrom and Laffont, 1979; Moskowitz and Vissing-Jørgensen, 2002; Landier and Thesmar, 2008; Hall and Woodward, 2010). Or there might be a *non-pecuniary benefit* from firm ownership that increases with wealth (Hurst and Pugsley, 2017).[6] In this spirit, Hamilton (2000) finds a "self-employment" discount, showing that entrepreneurship tends to persist despite lower earnings growth compared to paid employment (see also Pugsley and Hurst (2011) and Catherine (2022)), although Bhandari et al. (2022) challenge this result using their administrative data.

Our model demonstrates that the causal effect of wealth on firm profits can help to distinguish among these channels. The finding of higher profits suggests wealth increases

[4]See also De Mel et al. (2008) who show that wealth shocks are important for the performance of microenterprises in Sri Lanka.

[5]Buera (2009) notes that selection implies that the distribution of entrepreneurial ability among workers and entrepreneurs varies along the wealth distribution, but in the context of the relationship between wealth and entrepreneurship rather than the effect of wealth on business outcomes.

[6]An alternative explanation for entrepreneurs investing substantial equity and thus holding substantial non-diversified risk in their firms focuses on signaling incentives to lenders under asymmetric information, see Leland and Pyle (1977), Ross (1977), and Nenov (2017), among others.

entrepreneurship at least in part by relaxing financial constraints. Consistent with this interpretation, we also find a near one-to-one pass-through from wealth to equity in the firm and that stock market wealth matters less for entrepreneurship decisions of high financial wealth households. A finding that financial constraints matter in turn creates a possible role for development policies such as business subsidies that mitigate financial frictions (Itskhoki and Moll, 2019).

Our focus on the effect of stock market wealth for the real economy brings our paper close to recent work by Di Maggio et al. (2020), Chodorow-Reich et al. (2021), Andersen et al. (2021), and Ring (2022), among others. Unlike these papers, we consider the effects of stock market wealth for entrepreneurship and business creation rather than the effects on consumer spending (Di Maggio et al., 2020; Andersen et al., 2021; Chodorow-Reich et al., 2021) or the effects of wealth of existing firm owners on firm employment and investment during the 2008-2009 financial crisis (Ring (2022)). In independent work, Andersen et al. (2021) use a similar approach to isolating variation in returns by controlling for portfolio characteristics. We provide a formal justification for this approach by linking portfolio choice to the general problem of non-random exposure as in Borusyak and Hull (2021).

# 2 The Model and Testable Hypotheses

We start by developing the hypotheses that we test in our empirical analysis. Section 2.1 presents a simple static model of entrepreneurship choice as a function of wealth and business productivity. Section 2.2 formalizes the prediction that greater wealth increases entrepreneurship, which we test in Section 5. Section 2.3 introduces two microfoundations of why greater wealth increases entrepreneurship, one rooted in financial frictions and the other in non-pecuniary benefits of entrepreneurship, and shows that the causal effect of wealth on profits distinguishes them. Section 2.4 turns to how to estimate the causal effect of wealth on business outcomes such as profits. We show that endogenous entry into entrepreneurship generates selection on unobserved productivity even when wealth is randomly assigned and we develop a procedure to correct for this selection effect, which we implement in Section 6. Section 2.5 extends the basic model and the results in Section 2.4 to the important case where the entrepreneurship decision depends on additional dimensions of heterogeneity. Finally, Section 2.6 provides a dynamic extension in which positive portfolio returns increase entry, as in our empirical implementation.

## 2.1 Setup

The baseline model has a single period. There is a continuum of agents denoted by $i$ with mass normalized to one. Agents are associated with unobservable potential business productivity $z_i \in Z \subset R_+$, observable initial assets $a_i \in A \subset R_+$, and an observable vector of covariates $x_i \in X$ that might be correlated with productivity and assets. Let $F_{za}(z_i, a_i|x_i)$ denote the joint cumulative distribution function (CDF) of productivity and assets conditional on $x_i$. We assume the corresponding probability distribution function (PDF), denoted by $f_{za}(z_i, a_i|x_i)$, is continuous. We use a similar notation for other distributions. For instance, $F_z(z_i|a_i, x_i)$ denotes the CDF of $z_i$ conditional on $a_i$ and $x_i$, and $F_z(z_i|x_i)$ denotes the marginal CDF of $z_i$ conditional on $x_i$.

In the empirical analysis, we assume $a_i = a_{i,t-1}r_{i,t}$ and isolate quasi-random fluctuations in the portfolio return $r_{i,t}$ by controlling for covariates such as *ex ante* portfolio characteristics. Therefore, in the model we impose the following conditional independence assumption:

**Assumption (CIA).** $z_i$ and $a_i$ are independent *conditional on* $x_i$, that is: $F_z(z_i|a_i, x_i) = F_z(z_i|x_i)$ (and a similar condition holds for $F_a$).

An individual $i$ chooses whether to enter into business, $E_i \in \{0, 1\}$. If she does not enter, $E_i = 0$, she earns the outside option (reservation wage) $w(z_i)$. The reservation wage can depend on $z_i$, but we require this dependence to be relatively small in a sense that we formalize below. In this case, the individual's (consumption-equivalent) utility is equal to her assets plus her wage

$$U(E_i = 0) = a_i + w(z_i).$$

If instead the individual enters, $E_i = 1$, she runs a business with size $k_i = k(z_i, a_i)$ and earns profits $\pi_i = \Pi(k_i; z_i, a_i) = \pi(z_i, a_i)$. The individual's consumption-equivalent utility is

$$U(E_i = 1) = a_i + \pi(z_i, a_i) + u^e(z_i, a_i).$$

Here, the profit term captures the pecuniary benefit from entrepreneurship and the last term captures a possible non-pecuniary benefit—individuals might enjoy running their own business. An individual enters into business if her potential consumption-equivalent utility from entrepreneurship exceeds her potential wage

$$U(E_i = 1) \geq U(E_i = 0) \Longrightarrow \pi(z_i, a_i) + u^e(z_i, a_i) \geq w(z_i).$$

Observe that we assume the outside option $w$, size $k$, profit $\pi$, and entrepreneurship benefit $u^e$ depend *only* on productivity and initial wealth. Observed covariates $x_i$ affect these outcomes only through their impact on $z_i, a_i$ and other unobserved variables do not affect these outcomes. These assumptions are restrictive but they help to illustrate our results while simplifying the notation. In Appendix B.2, we show that under additional assumptions our results extend to cases in which the outcomes $w, k, \pi, u^e$ can be heterogeneous in other (observed or unobserved) dimensions. We discuss this extension in Section 2.5.

To characterize and estimate the effects of wealth on entrepreneurship, we impose mild monotonicity conditions on the profit, entrepreneurship benefit, and size functions.

**Assumption (M).** $\frac{d(\pi(z_i,a_i)+u^e(z_i,a_i))}{da_i} \geq 0$ and $\frac{d(\pi(z_i,a_i)+u^e(z_i,a_i)-w(z_i))}{dz_i} > 0$, $\frac{dk(z_i,a_i)}{dz_i} > 0$.

The first condition captures the main effect we investigate: greater wealth increases the total utility from entrepreneurship. This could be either because greater wealth increases profits by relaxing financial constraints, or because greater wealth increases the non-pecuniary benefits from entrepreneurship. We will shortly show how to distinguish among these explanations. The last two conditions enable us to address the selection effect induced by entry and estimate the effect of wealth on firm-level outcomes. The second condition says that business productivity increases the total utility from entrepreneurship faster than it increases the reservation wage (so it increases the net gain from entrepreneurship). The third condition says that individuals with higher productivity start larger businesses.

## 2.2 Effect of Wealth on Entry

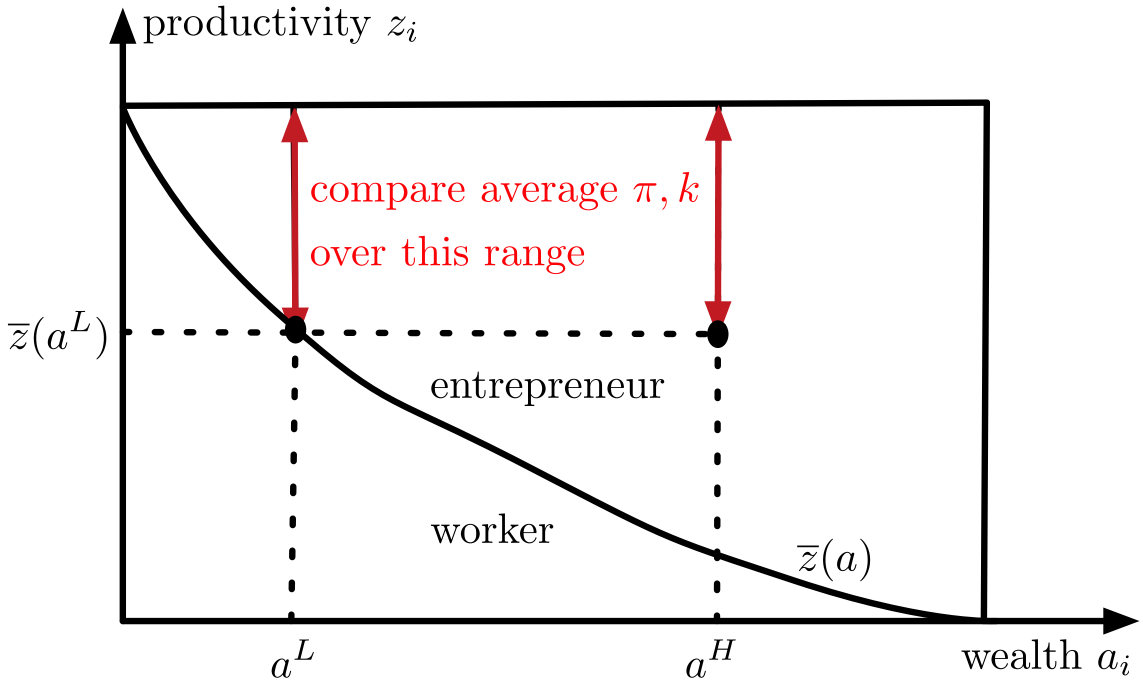
Our first main result concerns how initial wealth affects the entrepreneur's propensity to enter into business. To this end, we define the fraction of agents with assets $a$ that enter into business conditional on covariates

$$e\left(a|x_i\right) = \int_{(z_i,a)} E_i dF_{za}\left(z_i, a|x_i\right) \in (0,1)\,.$$

Proposition 1 shows that this fraction is increasing in initial assets: greater wealth increases the propensity to enter. This is the first hypothesis that we test in our empirical analysis.

**Proposition 1** (Causal effect of assets on entry)**.** *Consider the entry model with Assumption (M). Given $a$, there exists a threshold productivity level $\overline{z}\left(a\right)$ such that an agent enters iff $z \geq \overline{z}\left(a\right)$. The threshold productivity $\overline{z}\left(a\right)$ is* weakly decreasing *in initial assets. Higher assets induce higher entry into business: the fraction of entrants conditional on covariates, $e\left(a|x_i\right)$, is given by $\int_{z_i \geq \overline{z}(a)} dF_z\left(z_i|x_i\right)$, and it is weakly* increasing *in initial assets $a$.*

Figure 1: A Model of Endogenous Entry into Business




Notes: Individuals above the curved line, $\overline{z}(a)$, enter into business. We identify the causal effect of wealth on profits and capital by comparing the outcomes for individuals that would have entered both with low wealth $a_L$ and with high wealth $a_H$.

Under Assumption (M), individuals enter only if they have either sufficiently high wealth or sufficiently high productivity. Thus, there is a threshold productivity $\overline{z}(a)$, *decreasing in wealth*, such that individuals enter only if their productivity exceeds this level. This also implies that, conditional on covariates, greater wealth increases the propensity to enter. Figure 1 illustrates the threshold function $\overline{z}(a)$ and the region of entry in a particular example. Appendix B.4 contains proofs of all propositions.

## 2.3 Explicit Microfoundations and the Role of Profits

We now describe two example models that satisfy Assumption (M) and hence in which Proposition 1 holds (see Appendix B.1 for details). These models illustrate complementary mechanisms by which wealth might increase entrepreneurship, while making distinct predictions for how wealth affects profits, $\frac{d\pi(z_i,a_i)}{da_i}$. The first model features financial frictions but does not have non-pecuniary benefits from entrepreneurship, $u^e(z_i, a_i) = 0$. In that model, the entrepreneur needs to obtain financing to pay for the fixed cost of starting a business and for the capital expenditures. She can borrow funds from outside financiers, but outside financing is costly and these costs are increasing in the amount that the entrepreneur borrows. Higher initial wealth (internal funds) reduces the need for outside financing, which in turn raises the entrepreneur's potential profits, $\frac{d\pi(z_i,a_i)}{da_i} \geq 0$, firm scale, $\frac{dk(z_i,a_i)}{da_i} \geq 0$, and the total utility from entrepreneurship, $\frac{d(\pi(z_i,a_i)+u^e(z_i,a_i))}{da_i} \geq 0$.

The second model features no financial frictions but the non-pecuniary benefits from entrepreneurship are given by a function $u^e = U^e(k, c; z_i, a_i)$ where $k$ is the size of the business, $c = a_i + \Pi(k; z_i)$ is regular consumption, and $\Pi(k; z_i)$ denotes profits as a function of size. We assume $\frac{dU^e}{dc} > 0$, $\frac{dU^e}{dk} \geq 0$, $\frac{d^2U^e}{dcdk} \geq 0$ (along with standard regularity conditions). These assumptions capture the idea that individuals enjoy running a (larger) business, and more so when their regular consumption is higher. Individuals choose their business size $k$ to maximize $\Pi + U^e$. In this case, greater wealth increases the total utility from entrepreneurship as before, $\frac{d(\pi(z_i,a_i)+u^e(z_i,a_i))}{da_i} \geq 0$, but the effect works through the non-pecuniary benefits $u^e(z_i, a_i)$. Crucially, unlike in the financial frictions model, greater wealth *decreases* profits, $\frac{d\pi(z_i,a_i)}{da_i} \leq 0$. Intuitively, the non-pecuniary benefits from entrepreneurship induce a firm size beyond the profit-maximizing level, which reduces profits. We next develop Propositions 2 and 3 to estimate the effect of wealth on profits $\frac{d\pi(z_i,a_i)}{da_i}$ (as well as other business outcomes), which we will use (along with other findings) to differentiate between the two models.[7]

## 2.4 Effect of Wealth on Business Characteristics

Identifying the effect of wealth on the intensive margin of business outcomes is more complicated than the extensive margin choice because entry into business induces selection on *unobserved* productivity. To see this, consider the *average* firm profits given wealth (and covariates) and *conditional on entry*,

$$\begin{aligned} \pi(a|x_i) &= \mathbb{E}[\pi_i | x_i, a_i = a, E_i = 1] \\ &= \frac{\int_{z_i \geq \bar{z}(a)} \pi(z_i, a)\, dF_z(z_i|x_i)}{e(a|x_i)} \text{ where } e(a|x_i) = \int_{z_i \geq \bar{z}(a)} dF_z(z_i|x_i). \end{aligned}$$

Suppose we empirically estimate $\frac{d\pi(a|x_i)}{da}$. This does not necessarily identify $\frac{d\pi(z_i,a|x_i)}{da}$, because changing assets $a$ also affects the average productivity of entrants. In our model, increasing $a$ *reduces* the average productivity of entrants (see Figure 1).

Since we do not directly observe productivity, we cannot address this selection problem by controlling for productivity $z$. Instead, we observe the initial wealth $a$ for all individuals

[7] Both models satisfy the remaining two conditions in Assumption (M) when the reservation wage are constant (or only weakly increasing in productivity). This is because the total utility from entrepreneurship and the size of the business are both increasing with productivity—these are standard effects that continue to hold with financial frictions or with non-pecuniary utility from entrepreneurship.

and capital $k$ (or firm size) for the individuals that enter. We next show how to use these *observed* outcomes to match individuals by productivity and estimate firm-level outcomes.

The result relies on sorting the entrants by their size $k$—which is monotonically increasing with productivity in view of Assumption (M)—to match them according to their unobserved productivity. To formalize this idea, we define the fraction of individuals with wealth $a$ that enter into business *and whose size exceeds a cutoff level* $\overline{k}$ conditional on covariates:

$$e\left(a, k \geq \overline{k} | x_i\right) = \int_{z_i \geq \overline{z}(a), k(z_i, a) \geq \overline{k}} dF_z\left(z_i | x_i\right).$$

In principle, we can observe these fractions conditional on $x_i$ for any assets $a$ and threshold $\overline{k}$. We also let $y\left(z_i, a_i\right)$ denote a firm-level outcome that is observed and that can be described as a function of the entrant's productivity and initial wealth, such as profits $\pi\left(z_i, a\right)$ or size $k\left(z_i, a\right)$.

**Proposition 2** (Rank preservation and the causal effect of assets on firm-level outcome)**.** *Consider an entrant with covariates $x_i$, initial wealth $a^L$, size $k^L$, and firm-level outcome $y^L$, along with* unobserved *productivity $z \geq \overline{z}\left(a^L\right)$. Let $a^H > a^L$ denote a higher wealth level and $\overline{k} \geq k^L$ denote the unique solution to the following:*

$$e\left(a^H, k \geq \overline{k} | x_i\right) = e\left(a^L, k \geq k^L | x_i\right). \tag{1}$$

*Let $\overline{y}$ denote the outcome corresponding to the firm with higher initial wealth $a^H$ and the cutoff size $\overline{k}$. Then, $\overline{y} = y\left(z, a^H\right)$: that is, $\overline{y}$ is the firm-level outcome the entrant would have if she had higher initial wealth (and the same productivity). Thus, the difference*

$$\overline{y} - y^L = y\left(z, a^H\right) - y\left(z, a^L\right)$$

*identifies the causal effect of initial wealth on the firm-level outcome for an entrant with productivity $z$.*

We refer to condition (1) as *rank preservation*. To understand this condition, consider an entrant with covariates $x_i$, wealth $a^L$, and unobserved productivity $z$, and suppose we increase her wealth from $a^L$ to $a^H$. In view of assumption (M), this change would leave the entrant's *relative rank for size* unchanged. Intuitively, with either $a^L$ and $a^H$, the individuals with productivity $\tilde{z} \geq z$ also enter and have a greater size than the entrant (and the individuals with $\tilde{z} < z$ would either not enter, or they would enter and have a smaller size than the entrant). Importantly, since we observe the size by rank for each asset level (captured by the functions $e\left(a, k \geq \overline{k} | x_i\right)$), we can solve (1) and calculate the size that the entrant would have if she had higher initial wealth. This in turn enables us to estimate the causal effect of initial wealth on firm-level outcomes such as profits or size.

While we can in principle compute the fractions $e\left(a, k \geq \overline{k} | x_i\right)$ and implement Proposition 2 separately for each $x_i$, in practice this computation is not feasible because even large population data sets, such as the one we use, have a much smaller number of entrepreneurs. Therefore, we also develop a version of the proposition that uses *unconditional* fractions along with propensity score reweighting to control for the covariates $x_i$. To state the result, consider the fraction of entrepreneurs with wealth $a$ and minimum size $\overline{k}$ aggregated over all covariates:

$$e\left(a, k \geq \overline{k}\right) = \int_{x_i} \int_{z_i \geq \overline{z}(a), k(z_i, a) \geq \overline{k}} dF_z\left(z_i | x_i\right) dF_x\left(x_i | a\right).$$

Matching $e\left(a^L, k^L\right)$ with $e\left(a^H, k \geq \overline{k}\right)$ (with appropriate $\overline{k}$) will no longer control for productivity since agents with different wealth levels can be associated with different covariates (captured by $dF_x\left(x_i | a\right)$) and these covariates can be associated with different levels of productivity (captured by $dF_z\left(z_i | x_i\right)$). However, we can correct for these differences by appropriately reweighting the marginal distributions $dF_x\left(x_i | a\right)$, following the large literature on propensity score reweighting (see, e.g., Rosenbaum and Rubin (1983); DiNardo et al. (1996); Heckman et al. (1998); Hirano et al. (2003)).

Formally, for the lower wealth level $a^L$, we define the *reweighted* fraction of entrepreneurs:

$$e^*\left(a^L, k \geq \overline{k}\right) = \int_{x_i} \int_{z_i \geq \overline{z}(a^L), k(z_i, a^L) \geq \overline{k}} dF_z\left(z_i | x_i\right) \omega\left(x_i\right) dF_x\left(x_i | a^L\right) \tag{2}$$

$$\text{where } \omega\left(x_i\right) = \frac{dF_x\left(x_i | a^H\right)}{dF_x\left(x_i | a^L\right)} = \frac{dF_a\left(a^H | x_i\right)}{dF_a\left(a^L | x_i\right)} \frac{dF_a\left(a^L\right)}{dF_a\left(a^H\right)}.$$

The second line defines the propensity weights $\omega\left(x_i\right)$ and applies Bayes rule. These weights can be estimated from data since they rely *only* on observable variables and are defined over the full set of agents. Intuitively, the fraction $e^*\left(a^L, k \geq \overline{k}\right)$ overweights (resp. underweights) the agents with covariates $x_i$ that are relatively more common (resp. less common) among agents with higher wealth $a^H$. This reweighting makes the sample with $a^L$ comparable to the sample with $a^H$ in terms of the distribution of covariates. Consequently, a version of Proposition 2 applies with the reweighted distribution.

**Proposition 3** (Rank preservation with propensity score reweighting)**.** *Consider the entrants with initial wealth $a^L$, size $k^L$, and firm-level outcome $y^L$, along with* unobserved *productivity $z \geq \overline{z}\left(a^L\right)$. Let $a^H > a^L$ denote a higher wealth level and $e^*\left(a^L, k \geq \overline{k}\right)$ denote* *the propensity score reweighted fraction of entrants with lower wealth* $a^L$ *defined in* (2)*. Let* $\overline{k} \geq k^L$ *denote the unique solution to:*

$$e\left(a^H, k \geq \overline{k}\right) = e^*\left(a^L, k \geq k^L\right). \tag{3}$$

*Let* $\overline{y}$ *denote the outcome corresponding to the firm with higher initial wealth* $a^H$ *and the cutoff size* $\overline{k}$*. Then,* $\overline{y} = y\left(z, a^H\right)$*: that is,* $\overline{y}$ *is the firm-level outcome the entrant would have if she had higher initial wealth (and the same productivity). Thus, the difference*

$$\overline{y} - y^L = y\left(z, a^H\right) - y\left(z, a^L\right)$$

*identifies the causal effect of initial wealth on the firm-level outcome for an entrant with productivity* $z$*.*

This result shows how to estimate the causal effect for particular entrants (with a specific size and productivity). In the empirical analysis, we focus on the *average* causal effect over all entrants with some initial wealth. Formally, fix a wealth level $a^L$ and let $k^L$ denote the *lowest-size* firm corresponding to entrants with $a^L$, with productivity given by the entry threshold, $z = \overline{z}\left(a^L\right)$. Eq. (3) then becomes:

$$e\left(a^H, k \geq \overline{k}\right) = e^*\left(a^L\right) = \int_{x_i} \int_{z_i \geq \overline{z}(a^L)} dF_z\left(z_i|x_i\right) \omega\left(x_i\right) dF_x\left(x_i|a^L\right).$$

In particular, among the entrants with higher level of assets $a^H$, we find the lowest-size entrant that would have entered also with the lower level of assets $a^L$, after balancing the covariates with propensity score reweighting. Denote the cutoff size with $\overline{k} = k^H\left(a^L\right)$. We then calculate the average outcome variable for high-asset entrants *with sizes above the cutoff*

$$\mathbf{y}\left(a^H, k \geq k^H\left(a^L\right)\right) = \mathbb{E}\left[y_i | a_i = a^H, E_i = 1, k \geq k^H\left(a^L\right)\right]. \tag{4}$$

We also calculate the average outcome variable for low-asset entrants *after propensity-score reweighting*

$$\mathbf{y}^*\left(a^L\right) = \mathbb{E}^*\left[y_i | a_i = a^L, E_i = 1\right] \equiv \frac{\int_{x_i} \int_{z_i \geq \overline{z}(a^L)} y\left(z_i, a^L\right) dF_z\left(z_i|x_i\right) \omega\left(x_i\right) dF_x\left(x_i|a^L\right)}{e^*\left(a^L\right)}. \tag{5}$$

Using Proposition 3, it is then easy to check that comparing $\mathbf{y}\left(a^H, k \geq k^H\left(a^L\right)\right)$ and $\mathbf{y}^*\left(a^L\right)$ identifies the average causal effect among the entrants with high wealth $a_H$ and productivity

$z \geq \overline{z}\left(a^L\right)$.[8]

Figure 1 illustrates this approach. By considering the high-wealth entrants with size above the cutoff, we select individuals with relatively high productivity $z_i \geq \overline{z}\left(a^L\right)$. These individuals enter regardless of whether they start with wealth $a^L$ or $a^H$: they are not subject to the selection effect we mentioned earlier. Therefore, comparing their average outcomes identifies the average causal effect of wealth on firm-level outcomes. In our empirical analysis, we calculate the empirical counterparts to $(4-5)$ and report the difference.

## 2.5 Residual Heterogeneity

Underlying the rank preservation condition in Propositions 2 and 3 is an inversion of the ranking of observed business size $k$ to infer the ranking of unobserved productivity $z$. This inversion explains why we assume that $k$ depends only on $z$ and wealth $a$. In practice, $k$ can also be heterogeneous along other dimensions. For instance, *ex post* heterogeneity in size (or profits) could arise because an individual might be subject to productivity shocks after deciding to enter into business or because she might make a mistake (relative to the optimal choice). In addition, *ex ante* heterogeneity in size (or profits) can emerge as individuals might have industry-specific skills that imply variation in their prospective firms' production processes or startup costs, adding an additional argument to the functions determining $k$ (and $\pi$). Likewise, individuals' outside options $w$ or their utility from entrepreneurship $u^e$ might feature residual heterogeneity that is not fully captured by productivity $z$, e.g., due to differences in potential wage income and entrepreneurial productivity (or taste). These observations raise the question of how these types of unobserved residual heterogeneity affect our results.

In Appendix B.2, we show that our rank preservation approach is robust to allowing for ex-post and ex-ante residual heterogeneity under two additional assumptions. First, we require the residual heterogeneity to be independent from initial wealth and entrepreneurial productivity conditional on the observed covariates $x$. Second, we focus on entrants with size levels $k$ that exceed the entry cutoff for size by a sufficient margin.

The intuition for these conditions is as follows. While residual heterogeneity shuffles the firms' sizes, it does not necessarily bias our approach in a particular direction when it is conditionally independent from initial wealth and productivity. Some less productive firms

[8] In particular, we have

$$\mathbf{y}\left(a^H, k \geq k^H\left(a^L\right)\right) - \mathbf{y}^*\left(a^L\right) = \frac{\int_{x_i}\int_{z_i \geq \overline{z}(a^L)}\left(y\left(z_i, a^H\right) - y\left(z_i, a^L\right)\right) dF_z\left(z_i|x_i\right) dF_x\left(x_i|a^H\right)}{e^*\left(a^L\right)}.$$

become larger and some more productive firms become smaller, but on average the firms with a given size $k$ have a similar productivity as in the baseline without residual heterogeneity. In fact, under appropriate technical assumptions, the model satisfies certainty-equivalence properties that enable us to extend our main result to the case with residual heterogeneity as long as we focus on individuals that are sufficiently far from the entry cutoffs (see Proposition 4).[9]

For individuals close to the entry cutoffs, our approach is not necessarily robust to residual heterogeneity, because of a selection problem induced by the interaction of unobserved residual heterogeneity with the entry decision. Some individuals that would choose size $k$ have relatively low productivity (and high residual-induced size) so they might choose not to enter. Therefore, *the entrants* with size $k$ might have a higher average productivity than in the baseline case. Since this selection might be different for the control group of individuals with lower initial wealth and the test group with higher initial wealth, our rank preservation approach does not necessarily control for the average productivity between these two groups.

We alleviate this concern in a robustness exercise where we focus on entrant firms whose size exceeds the entry cutoff by some margin. For a sufficient margin (that depends on the extent of unobserved residual heterogeneity), these firms are not subject to the selection concern driven by entry (see Appendix B.2). We also present additional robustness exercises that we describe in Section 6.3 that address certain identifiable forms of heterogeneity. Finally, it merits emphasizing that Proposition 1 does not depend on the inversion property. As a result, it goes through with any form of heterogeneity as long as Assumption (CIA) holds.

## 2.6 Dynamic Extension and the Role of Portfolio Returns

So far we have worked with a static model to highlight the main predictions and to show how to correct for the selection on unobserved productivity on firm outcomes. In Appendix B.3 we develop a multi-period extension that more directly motivates our empirical framework. Focusing on the case with financial frictions, we characterize the dynamic decisions of a worker whose wage $w$ remains constant over time and an entrepreneur whose business productivity $z$ remains constant over time. We then characterize the choice between staying a worker and entering into business.

[9] We can characterize the consequences of certain forms of heterogeneity correlated with ability. A *positive* correlation of ability and preference for larger size tends to "pull apart" the size distribution, leaving the rank preservation property intact such that our results continue to hold. If instead ability correlates *negatively* with preference for larger size, then some larger firms in the high wealth group will have owners of lower ability than in the low wealth group. This would bias *against* finding that higher wealth causes higher profits.

As in the static model, the individual enters into business if her wealth exceeds a cutoff, $a_t \geq \overline{a}(z)$, where the cutoff is decreasing in $z$. The difference is that the individual's wealth evolves endogenously driven by her savings and portfolio decisions. Under simplifying assumptions, if the individual chooses to work, then her wealth evolves according to (see Eq. $(B.32)$):

$$a_{t+1} = \left(\beta a_t + w\left(\frac{\beta - 1/r^f}{1 - 1/r^f}\right)\right) r_{t+1}.$$

Here, $\beta$ denotes the discount factor that captures the individual's propensity to save, $r^f$ is the risk-free return, and $r_{t+1}$ is the individual's (endogenous) portfolio return. Combining these wealth dynamics with the entry decision, the individual enters into business in period $t$ as long as the following two conditions hold (see Eq. $(B.31)$):

$$a_t < \overline{a}(z) \text{ and } r_{t+1} \geq \overline{r}\,(z, a_t) \equiv \frac{\overline{a}\,(z)}{\beta a_t + w\left(\frac{\beta - 1/r^f}{1 - 1/r^f}\right)}.$$

The second condition $r_{t+1} \geq \overline{r}\,(z, a_t)$ says that the agent becomes an entrepreneur if her realized portfolio return exceeds a cutoff. This condition highlights that the entry decision is driven by wealth *changes* and motivates our regression specification with the portfolio return as the independent variable that we adopt in the next section.

The first condition $a_t < \overline{a}(z)$ says that entrants have wealth below their cutoff value in the last period and experience an *increase* in their wealth when they enter. Intuitively, would-be entrants with a higher wealth had already entered in past periods. This condition suggests that the portfolio return is likely to have an asymmetric effect on entry, with positive returns generating stronger effects than negative returns (see the Appendix for a formalization). We explore this type of asymmetry in the effect of return in our empirical analysis.

# 3 Data and Definitions

We combine a number of administrative data sets from Norway using unique personal and firm identification numbers as well as the unique ISINs of publicly traded shares. The unit of analysis is a household. Our data set construction shares similarities with Fagereng et al. (2020) and Ring (2022).

## 3.1 Data

We obtain information on the composition of stock portfolios and business ownership from the shareholder register ("Aksjonærregisteret"). This data set records information on ownership of shares in Norwegian limited liability firms, both publicly traded and privately held, and the book value of those shares at the end of each calendar year starting in 2004. The information is collected by the Norwegian tax authority and is third-party reported by financial intermediaries and includes stocks held in individual retirement accounts.[10] Using the security-level ISIN numbers for publicly traded stocks, we merge the stock ownership data with prices and returns for all publicly traded stocks on the Oslo stock exchange (OSE). These returns account for stock splits and other similar events, allowing us to construct the buy-and-hold market return on the household's portfolio of Norwegian stocks.[11] We use the shareholder register information on shares in privately held companies to determine entrepreneurship, as discussed further below.

We obtain household balance sheet and income information from tax records ("Inntekt" register). The household balance sheet information includes total gross financial wealth subject to the Norwegian wealth tax and asset holdings for broad asset classes such as deposits, publicly-traded Norwegian stocks, stock and bond mutual funds, bonds, and foreign assets. It also comes from third-party reporting to the Norwegian tax authority (except ownership of foreign assets). We do not know the details of specific asset holdings within broad asset classes of financial wealth outside of publicly-traded Norwegian stocks, so the variation in portfolio returns will come only from the Norwegian stock component. However, being able to quantify the "known unknown" in financial wealth will prove important in our research design.

The Norwegian register data also provide a number of variables used as covariates in the analysis, including education and age of the household members, family status, and municipality of residence ("Befolkning" and "Utdanning" registers). We obtain the NACE sector of primary employment of the highest earning individual in the household ("household head") by merging the tax records to the employer-employee register ("Aa-registeret").

Our firm-level data start with information from the "Aksjonærregisteret" on all limited liability firms in Norway, including the exact foundation date, closing date (if the firm is dissolved), primary sector, the total number of shareholders for different classes of shares, and the book value of outstanding shares for each firm. We combine with information on employment from the employer-employee register, as well as annual firm balance sheets and

[10]Most directly-held stock wealth in Norway is held outside of individual retirement accounts, as such accounts were not particularly wide-spread for most of our sample. Smogeli and Halvorsen (2019) report that in 2017 the aggregate value of "Individuell Pensjonssparing" (IPS) accounts was around 37 billion NOK, or 0.7% of total retirement wealth in Norway. Around 353 thousand people aged 17 and over had such retirement accounts, with a median balance of 48 thousand NOK. The vast majority of retirement wealth is in the national social insurance fund, with a smaller amount managed by occupational pensions.

[11]See Ødegaard (2013) for details of the OSE data.

income statements from tax records ("Regnskapsdata").[12] We use the employment in the subsequent year in the cases when employment in year of foundation is missing.

We restrict attention to households with a household head between 20 and 65 years old. We drop household-year observations with no earnings and zero financial wealth, as well as observations with (lagged) real gross financial wealth below 50 thousand and above 5 million Norwegian kroner. We also drop observations of direct stock owners with less than 1% (lagged) exposure to domestic publicly traded stocks. Furthermore, we drop from our sample households after they become entrepreneurs. Our final sample covers the period 2004-2019.

## 3.2 Entrepreneur Definition

We define an entrepreneur as an individual who owns at least 1/3 of the book value of stocks in an incorporated non-financial firm, and where the firm has at most 3 stock owners and at least one employee in the year of foundation or subsequent year. We further require that either the newly-created firm does not own publicly traded domestic stocks in the year of foundation or that it has employees that are not members of the entrepreneur's household. A transition to entrepreneurship requires both a newly-created firm and that the household not have owned stocks in any private firm in the past. Upon transitioning to entrepreneurship, a household exits our sample.

These restrictions collectively focus attention on first-time active owners of new firms.[13] In particular, the limit on number of owners helps to exclude passive investment positions in private firms (e.g. angel investing), the employment restriction ensures the new firm is economically active, and the requirement that either the firm have employees unrelated to the entrepreneur or that the firm holds no public equity helps to filter out inactive "family investment firms" created to store unrealized capital gains or losses for tax simplification purposes.

To put our entrepreneurship definition into perspective, Table 1 reports shares of business ownership and transitions to different types of business ownership both among the owners of publicly traded Norwegian stocks ("Stock owners"), as well as for our whole sample ("Population"). The first row includes both owners of at least 1/3 of the book value of any incorporated firm as well as households that receive non-incorporated business or farm income. We then progressively tighten the definition until we arrive at the definition of

[12] We deflate all nominal values and returns to 2010 Norwegian kroner. Throughout our sample period the dollar-kroner exchange rate fluctuates between 4.9 and 9.3, with a mean of 6.8.

[13] See Brandt et al. (2022) for an analysis of the differences between serial and non-serial entrepreneurs using detailed firm-level data from China.

entrepreneurship we use.[14]

In addition to this summary statistics table, Appendix Table A.4 includes additional descriptive statistics for the groups of entrepreneurs and non-entrepreneurs, as well as the groups of direct owners of Norwegian public stocks and the rest. Entrepreneurs tend to be younger, with slightly higher earnings but slightly lower financial wealth and lower holdings of publicly traded domestic stocks compared to non-entrepreneurs. Direct owners of domestic publicly traded stocks constitute around 12.6% of our sample. These households tend to be older, have higher earnings and higher level of financial wealth than the rest of our sample. The fact that entrepreneurs have higher earnings upon transitioning compared to non-entrepreneurs but lower direct holdings of domestic publicly traded stocks is broadly in line with the findings of Bhandari et al. (2022) using data from the U.S.

Table 1: Business Owners Descriptive Statistics

| | Stock owner (in %) | Population (in %) |
|---|---|---|
| **(I)** owns $\geq \frac{1}{3}$ book value (BV) of an incorporated firm or receives business/farm/forestry income | 19.11 | 17.75 |
| **(II)** owns $\geq \frac{1}{3}$ BV of an incorporated firm ("owns a business") | 6.07 | 4.98 |
| **(III)** owns a business with $\leq 3$ shareholders | 4.95 | 4.22 |
| **(IV)** AND is non-financial firm | 4.39 | 3.83 |
| **(V)** AND has employees | 2.32 | 2.27 |
| **(VI)** transitions to a business such as in (V) | 0.87 | 0.82 |
| **(VII)** transitions to a business such as in (V) that is newly created and not "family investment firm" | 0.18 | 0.21 |

Notes: The table reports shares of business ownership and transitions to different types of business ownership both among the owners of publicly traded Norwegian stocks ("Stock owners") as well as for our whole sample ("Population"). The first row includes both owners of at least 1/3 of the book value of any incorporated firm as well as households that receive non-incorporated business or farm income. Subsequent rows progressively narrow this group as described in the first column of the table. Note that our baseline definition for entrepreneurs includes those in (VII) but for whom the transition happens for the first time. Therefore, in principle the definition in (VII) and our baseline definition of entrepreneurship do not overlap exactly because of serial entrepreneurship with breaks in the data. In practice, however, the difference is negligible as a share of the population and the shares in row (VII) essentially coincide with the shares of entrepreneurs as per our baseline definition.

[14]Strictly speaking our baseline definition for entrepreneurs includes those in (VII) but for whom the transition happens for the first time. Therefore, in principle the definition in (VII) and our baseline definition of entrepreneurship do not overlap exactly because of the possibility of serial entrepreneurship with breaks. In practice, however, the difference is negligible as a share of the population, and so the shares in row (VII) essentially overlap with the shares of entrepreneurs according to our definition.

# 4 Econometric Methodology

This section presents our baseline specification and explains how it addresses the main threats to causal identification.

## 4.1 Econometric Design

Let $E_{i,t}$ denote an indicator for individual $i$ becoming an entrepreneur in year $t$. We model $E_{i,t}$ as a function of the return on financial wealth $r^*_{i,t}$, other *ex ante* observed characteristics $X_{i,t-1}$, and unobserved characteristics $\epsilon_{i,t}$.[15] The terms $X_{i,t-1}$ and $\epsilon_{i,t}$ include determinants discussed in Section 2 such as baseline financial wealth, the wage if remaining in paid employment, and entrepreneurial ability, as well as other factors such as preferences.

We do not observe $r^*_{i,t}$, because while we observe total financial wealth at the end of each year we do not observe all transactions during the year. We therefore focus on the buy-and-hold return from the direct stock portfolio that we can measure precisely. Formally, we decompose the total wealth return as

$$r^*_{i,t} = r_{i,t} + \epsilon^r_{i,t} \text{ where } r_{i,t} = r^f_t + \omega_{i,t-1}\mathbf{s}'_{i,t-1}\mathbf{r}_t,$$

where $r^f_t$ is the risk-free return, $\omega_{i,t-1}$ denotes the share of financial wealth held in domestic stocks at the end of year $t-1$, $\mathbf{s}_{i,t-1}$ the weight in the stock portfolio of each domestic stock, and $\mathbf{r}_t$ the vector of realized excess returns of domestic stocks in year $t$. The buy-and-hold return $r_{i,t}$ reflects the excess returns from the stock portfolio assuming the individual holds these stock positions throughout year $t$. The residual return $\epsilon^r_{i,t}$ captures the excess returns from other risky assets as well as the excess returns from the trading of stocks in year $t$. Below, we explain how we isolate quasi-random variation in $r_{i,t}$ by including a suitable array of fixed effects. Under this condition, a regression of the total wealth return $r^*_{i,t}$ on $r_{i,t}$ and the same fixed effects would yield a coefficient of one, reflecting the restriction to variation in $r_{i,t}$ from quasi-random realizations of idiosyncratic risk that is uncorrelated with $\epsilon^r_{i,t}$. We therefore impose this "first stage" coefficient of one and directly model outcomes in terms of $r_{i,t}$. Note that this instrument is stronger when the individuals' stock portfolios are more persistent over time. Empirically, we find stock portfolios are quite persistent, with a large share of stock holders not adjusting their portfolio over a one year horizon (see Table A.2).

[15] Our baseline model in Section 2 relates $E_{i,t}$ to assets rather than returns. Our dynamic extension in Section 2.6 shows that using the returns is equivalent if all individuals start at the same asset level. We work in the return space because this avoids a mechanical relationship between higher initial assets and being in extreme bins of changes in assets. In addition to interpreting the marginal effect of a higher return at the median asset level, we estimate a specification in level changes in robustness.

Two main threats to causal identification remain. To frame them, it helps to decompose the stock portfolio excess return into systematic and idiosyncratic components: $\mathbf{s}'_{i,t-1}\mathbf{r}_t = \beta_{i,t-1} \times r^m_t + \nu_{i,t}$, where $r^m_t$ is the market excess return in year $t$, $\beta_{i,t-1}$ is the portfolio "beta" for stock holdings at end of $t-1$, and $Var(\nu_{i,t}) = \sigma^2_{i,t-1}$.[16] The first threat arises because the *realized* idiosyncratic component $\nu_{i,t}$ may be correlated with unobserved determinants of entrepreneurship $\epsilon_{i,t}$. For example, home bias in portfolio choice (Coval and Moskowitz, 1999) could result in households experiencing better stock market returns in periods when their current industry or local area is booming. We address such concerns by including sector×time and municipality×time fixed effects in all specifications.

The second threat arises because *expected* returns may vary across households in a manner correlated with the entrepreneurship decision. For example, a more risk-tolerant individual might choose a stock portfolio with a higher market beta, implying higher expected returns, and risk tolerant individuals might also be more likely to transition to entrepreneurship for other reasons. Or individuals likely to become entrepreneurs might hold more or less of their wealth in domestic stocks. Borusyak and Hull (2021) term this "non-random exposure" and show that in linear regression it suffices to control for the *ex ante* expected realization. In our setting, variation in exposure comes from portfolio characteristics. Specifically, the expected total buy-and-hold portfolio return is $\mathbb{E}_{t-1} r_{i,t} = \mathbb{E}_{t-1} r^f_t + \omega_{i,t-1} \times \beta_{i,t-1} \times \mathbb{E}_{t-1} r^m_t$.

We control flexibly for different expected returns by creating bins of $\omega_{i,t-1}, \beta_{i,t-1}$, and $\sigma_{i,t-1}$ and including interactions of these bins and time fixed effects, where the interactions with time accommodate unrestricted time-variation in the risk-free rate or expected market return. The non-parametric controls for portfolio characteristics and inclusion of the return variance $\sigma_{i,t-1}$ is necessary for the non-parametric and non-linear specifications reported below.[17] Effectively, we compare entrepreneurship rates across two individuals in the same year with the same allocation to domestic stocks and the same portfolio beta but different realized excess returns. The variation in $r_{i,t}$ thus comes from random realizations on portfolios with the same *ex ante* characteristics, where the randomness arises as the result of the idiosyncratic component of the portfolio holdings $\nu_{i,t}$ purged of industry or location characteristics.

To summarize, our baseline specification for the effect of the stock market on transitioning

[16] We use this timing notation because we hold fixed the characteristics $\beta_{i,t-1}$ and $\sigma^2_{i,t-1}$ at their values from the previous year. We omit expected excess returns ("alpha") from the return representation because we find that the idiosyncratic component of returns has a small and *negative* serial correlation in our data.

[17] The bins for $\omega$, $\beta$, and $\sigma$ should have relatively low within-bin dispersion. We therefore truncate their distributions at the 2nd and 98th percentiles and report robustness in Section 5.2 to not truncating.

to entrepreneurship takes the form:

$$E_{i,t} = b \times r_{i,t} + \alpha_{\text{sector}\times t}(i) + \alpha_{\text{munic.}\times t}(i) + \alpha_{\beta\times\sigma\times\omega\times t}(i) + \epsilon_{i,t}, \tag{6}$$

where $\alpha_{y\times z}(i)$ denotes a fixed effect for observation $i$ belonging to group $y \times z$. In some specifications we also control for additional covariates. These covariates absorb residual variation in the likelihood of becoming an entrepreneur and relax specific assumptions in our baseline implementation.

## 4.2 Implementation

We measure the portfolio characteristics $\beta_{i,t-1}$ and $\sigma_{i,t-1}$ using daily returns over the year $t-1$. Specifically, for each observation we form the time series of daily returns $\mathbf{s}_{i,t-1}\mathbf{r}_{t-1+\Delta}$, where $\mathbf{r}_{t-1+\Delta}$ gives the vector of individual stock returns on day $t-1+\Delta$, and we fix the weights at their value at the end of the year.[18] We obtain $\beta_{i,t-1}$ as the OLS regression coefficient from a regression of $\mathbf{s}_{i,t-1}\mathbf{r}_{t-1+\Delta}$ on $r^m_{t-1+\Delta}$ (which we equate with the OSE OBX stock market index) and $\sigma_{i,t-1}$ as the variance of $\mathbf{s}_{i,t-1}\mathbf{r}_{t-1+\Delta}$.[19] Figures A.1-A.3 in the Appendix plot the distribution of $\beta$, $\sigma$ and $\omega$.

Figure 2 plots the unconditional distribution of (one year) buy-and-hold returns on financial wealth among the direct owners of publicly traded stocks in our sample.[20] Portfolio returns tend to be small on average but with a substantial standard deviation of around 12% and non-zero mass of relatively large return realizations of above 25%. The distribution of returns is also right-skewed, reflecting the in-sample positive mean return on the aggregate stock market, which we decompose in Figure 3 by splitting buy-and-hold portfolio returns in two groups based on whether the average return on the OSE OBX stock market index is above or below the 2004-2019 median. In our empirical analysis we will examine the heterogeneous effects of higher returns in years with above or below median aggregate stock market returns, which we will refer to as "good" and "bad" stock market years.

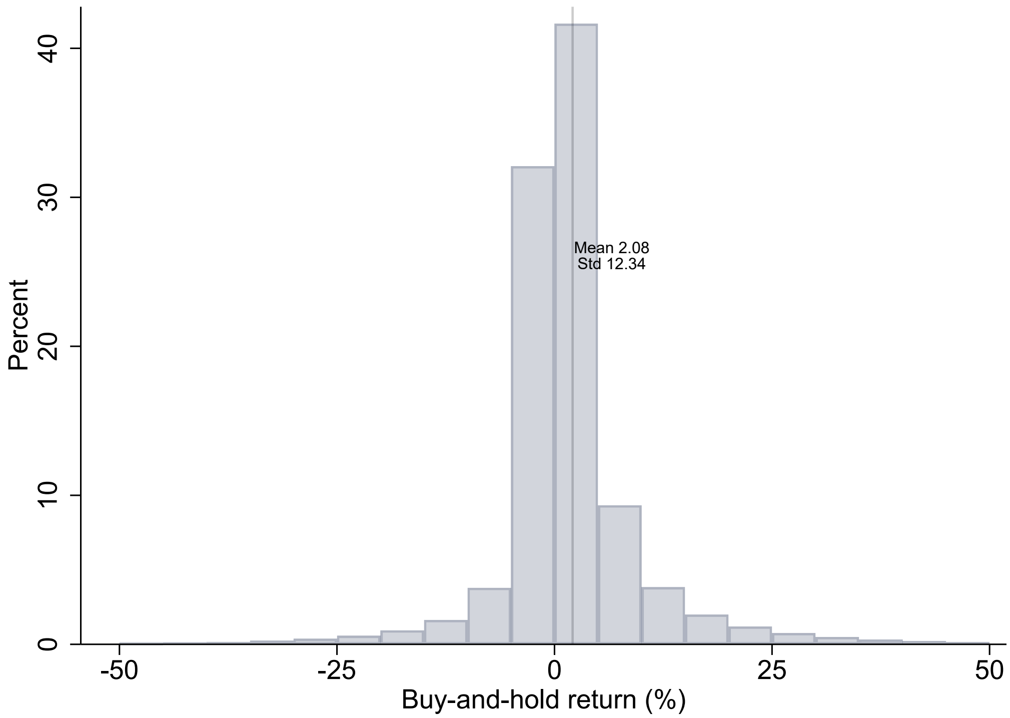
Where does variation in returns across households come from? Table A.2 reports statistics from the distribution of household portfolio characteristics. Portfolios of domestic stocks exhibit high concentration, with the median stock holder holding just two stocks. Such high

[18] We use price returns to focus on the unexpected component of the stock return but our results are little changed if we use total returns instead.

[19] Section 5.2 reports robustness to allowing for up to three portfolio factors or to replacing the OSE OBX with the U.S. CRSP value-weighted index, as well as several other robustness exercises that try to account for possible additional differences in portfolio characteristics that correlate with the propensity to start a firm.

[20] For households who are not direct owners, buy-and-hold returns will be set to zero, and these households will be treated as a separate category in our regressions.

Figure 2: Portfolio Return Distribution




Notes: Buy-and-hold returns $r_{i,t}$ are defined as $r_{i,t} = r_t^f + \omega_{i,t-1}\mathbf{s}'_{i,t-1}\mathbf{r}_t$, where $r_t^f$ is the risk-free return in year $t$, $\omega_{i,t-1}$ denotes the share of financial wealth held in domestic stocks at the end of year $t-1$, $\mathbf{s}_{i,t-1}$ the weight in the stock portfolio of each domestic stock, and $\mathbf{r}_t$ the vector of realized excess returns of domestic stocks in year $t$.

concentration implies an absence of diversification, making possible sizable idiosyncratic differences in returns. Direct stock owners tend to have a relatively limited investment in other risky assets such as stock mutual funds with the remaining share of financial wealth held in deposits. Additionally, portfolios tend to be quite sticky, with a large share of households not making any portfolio adjustments over a one year horizon.

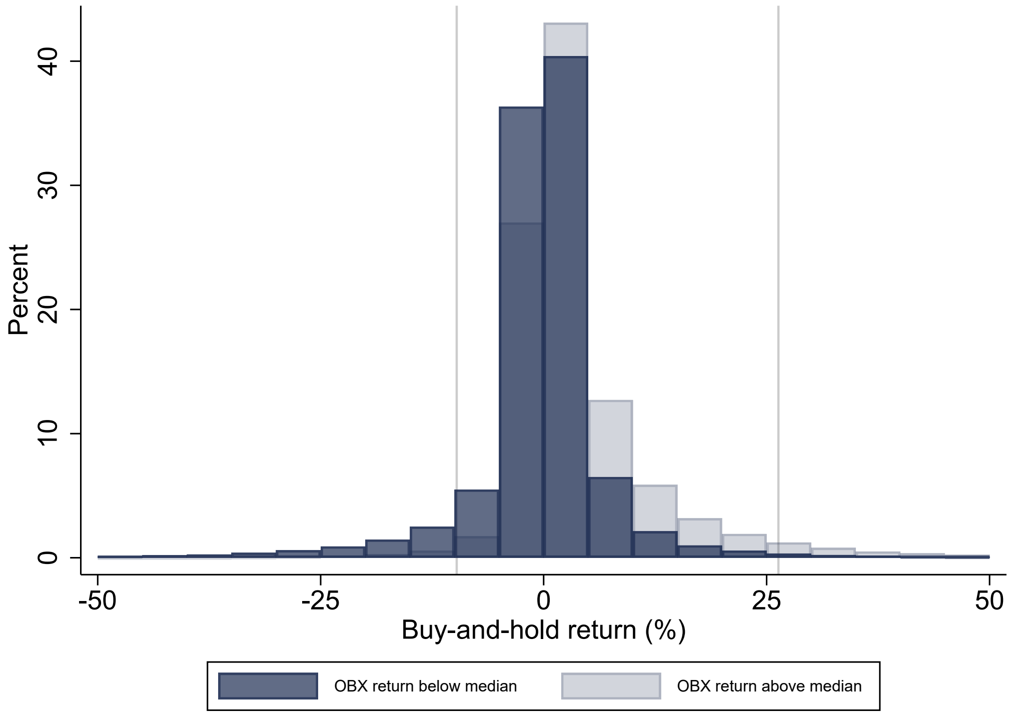
# 5 Results on the Propensity to Start a Business

In this section we present our main results on the effects of stock market wealth on the propensity to start a business.

## 5.1 Baseline Results

We start with a non-parametric approach. We partition the space of financial wealth and buy-and-hold portfolio returns into several bins and estimate the average effect from being

Figure 3: Portfolio Return Distribution: Good vs. Bad Stock Market Years




Notes: Buy-and-hold returns $r_{i,t}$ are defined as $r_{i,t} = r_t^f + \omega_{i,t-1}\mathbf{s}'_{i,t-1}\mathbf{r}_t$, where $r_t^f$ is the risk-free return in year $t$, $\omega_{i,t-1}$ denotes the share of financial wealth held in domestic stocks at the end of year $t-1$, $\mathbf{s}_{i,t-1}$ the weight in the stock portfolio of each domestic stock, and $\mathbf{r}_t$ the vector of realized excess returns of domestic stocks in year $t$. The figure plots the distributions of buy-and-hold returns for two groups of years: years in which the aggregate OBX return is above the median value in the period 2004-2019 and years in which the OBX return is below the median value.

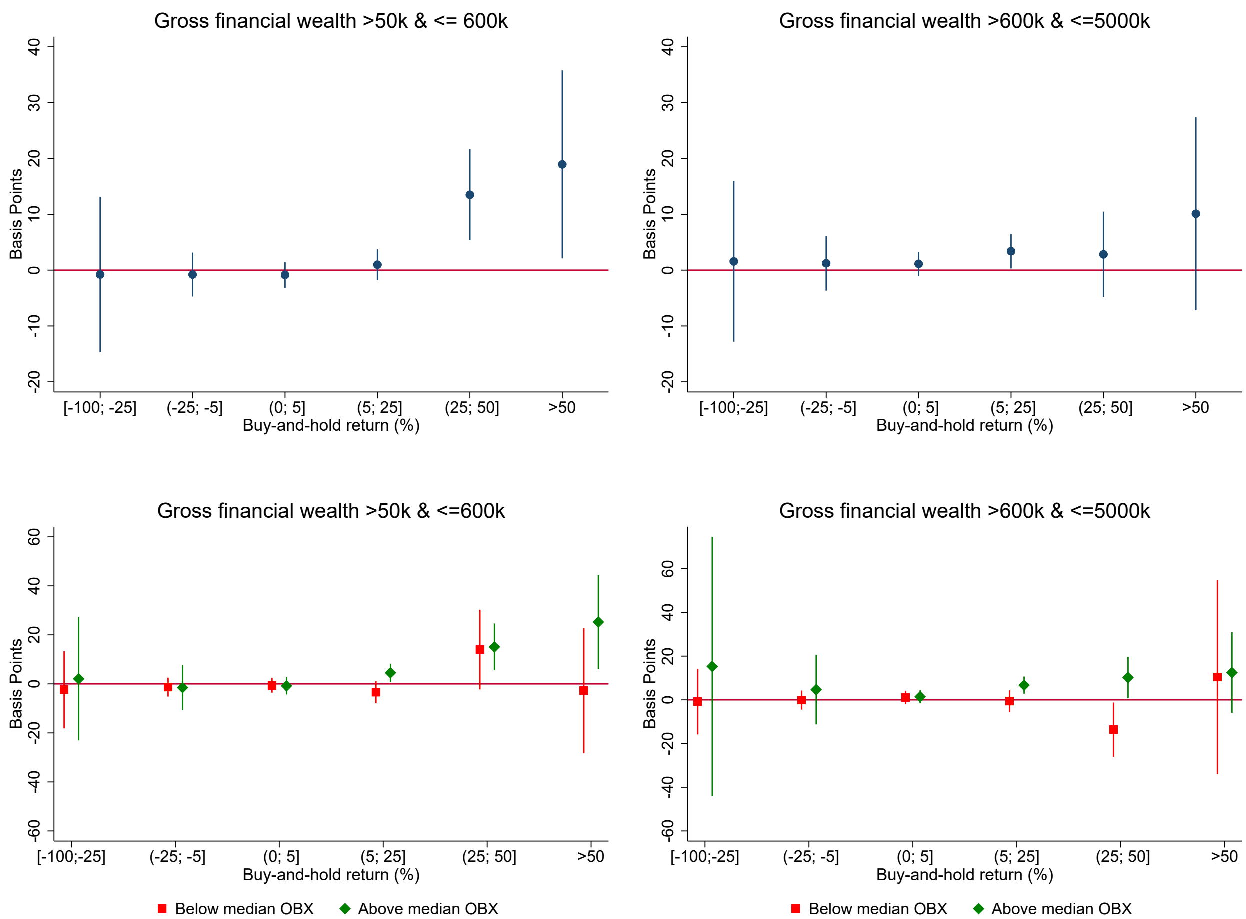
in a particular bin on the propensity to start a business relative to a specific base bin. Our specification includes the fixed effect controls described in Section 4. We split financial wealth in two groups: below 600k ("moderate wealth") and above 600k ("high wealth"). For buy-and-hold returns we have 7 bins with the return bin of (-5%, 0%] serving as base.[21]

The first row of Figure 4 presents the estimated relative effects for each wealth bin. There is a notable positive effect of having a relatively high return of 25% or above for the moderate wealth group. In contrast the effects for high wealth are much smaller. The effect also appears to be asymmetric in the sign of buy-and-hold returns with only significant effects observed for sufficiently high positive returns. This asymmetry is consistent with only positive returns moving individuals above their wealth cutoff for starting a business, as in the dynamic extension discussed in Section 2.6.

We also examine aggregate return heterogeneity by interacting each of the wealth-by-

[21] We also include an additional bin for households who are not direct domestic stock owners. Alternative thresholds for the moderate wealth group around 600k NOK deliver similar estimated effects.

Figure 4: Non-parametric Entry Results




Notes: We define an entrepreneur as an individual that owns more than 1/3 of the book value of stocks in an incorporated non-financial firm with at most 3 stock owners which employs at least one worker. When considering the transition to entrepreneurship we only consider newly-created firms and households who have not owned stocks in any private firm in the past. In addition we require that the newly-created firm does not own publicly traded domestic stocks in the year of foundation unless it has employees that are not members of the entrepreneur's household. Furthermore, upon transitioning to entrepreneurship a household is dropped from our sample. Effects in each return bin are relative to a base buy-and-hold portfolio return between -5% and 0%. The second row shows effects in years when the OBX index return is above or below the median for the period 2004-2019. Controls include age group indicators (for 3 age groups), municipality-by-year fixed effects, primary employment sector-by-year fixed effects, and a set flexible controls given by a four-way interactions between interaction between 8 bins of exposure to directly-held domestic stocks, 7 bins of betas for the observed part of the portfolio, 7 bins of volatility for the observed part of the portfolio and year. Buy-and-hold returns $r_{i,t}$ are defined as $r_{i,t} = r_t^f + \omega_{i,t-1}\mathbf{s}'_{i,t-1}\mathbf{r}_t$, where $r_t^f$ is the risk-free return in year $t$, $\omega_{i,t-1}$ denotes the share of financial wealth held in domestic stocks at the end of year $t-1$, $\mathbf{s}_{i,t-1}$ the weight in the stock portfolio of each domestic stock, and $\mathbf{r}_t$ the vector of realized excess returns of domestic stocks in year $t$. The bars represent 95% confidence intervals computed with clustering on the level of the municipality.

return bin with whether the aggregate stock return is above or below its median. The second row of Figure 4 shows the estimated relative effects (again with the (-5%, 0%] bin serving as base). There is a clear heterogeneity across good and bad stock market years. In bad stock market years the effect of returns is fairly flat and close to zero. In contrast, in good stock market years the propensity to start a business is increasing in return bins, particularly for the moderate wealth stock owners.

Table 2 reports regression coefficients. Column (1) pools the full sample and all years. The coefficient of 0.10 translates into a 2 basis point higher transition rate into entrepreneurship following a 20% stock return. Motivated by the evidence in Figure 4, column (2) restricts the sample to households with less than 600k, and column (3) allows the coefficient to vary between the moderate and high wealth groups. The coefficient estimate is larger for the sample excluding high wealth households and is also larger when comparing moderate to high wealth households. In particular, a two sided t-test rejects equality of the coefficients for moderate versus high wealth households at a significance level of 5%. Furthermore, column (4) shows that the effects are significantly larger for good stock market years compared to bad stock market years among the moderate wealth group. A two-sided t-test rejects equality of the coefficient estimates for a good versus bad stock market year in column (4) at a significance level of 1%. Finally, column (5) displays the asymmetric effects of positive versus negative buy-and-hold returns on the propensity to start a business, although the standard errors are too large to formally reject equality.

To put these magnitudes in context, the estimated effect in column (4) implies that a 20 p.p. increase in (one year) portfolio returns increases the propensity to start a business in a good stock market year by around 6.4 basis points, which is around 1/3 of the baseline entrepreneurship rate. Furthermore, the median financial wealth for the moderate wealth group of direct stock owners is close to 176k NOK, so a return of 20% corresponds to an increase in wealth for a household in the moderate wealth group with median financial wealth of 35k NOK, or approximately 5k USD based on the sample average exchange rate.[22]

[22]For further context, Norway has a minimum startup equity threshold that fell from 100k NOK to 30k NOK in 2012. We do not have power to detect different marginal responses before and after. See Bacher et al. (2024) for analysis of the effects of the threshold change on overall entrepreneurship rates.

Table 2: Baseline Entry Results

| | Dep. var.: becomes entrepreneur ($E_{i,t}$) | | | | |
|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) |
| $r_{i,t}$ | 0.109* | 0.226** | | | |
| | (0.045) | (0.073) | | | |
| $r_{i,t}$, gross fin. wealth $> 600k$ NOK | | | 0.023 | | |
| | | | (0.042) | | |
| $r_{i,t}$, gross fin. wealth $\leq 600k$ NOK | | | 0.170** | | |
| | | | (0.061) | | |
| $r_{i,t}$, $r_{OBX,t} \leq r_{OBX,median}$ | | | | −0.020 | |
| | | | | (0.102) | |
| $r_{i,t}$, $r_{OBX,t} > r_{OBX,median}$ | | | | 0.321** | |
| | | | | (0.090) | |
| $r_{i,t}$, $r_{i,t} \leq 0$ | | | | | −0.093 |
| | | | | | (0.212) |
| $r_{i,t}$, $r_{i,t} > 0$ | | | | | 0.280** |
| | | | | | (0.093) |
| Sample | All | $\leq 600k$ | All | $\leq 600k$ | $\leq 600k$ |
| Location-year FE | Yes | Yes | Yes | Yes | Yes |
| Primary sector-year FE | Yes | Yes | Yes | Yes | Yes |
| Portfolio-year FE | $\omega$-$\beta$-$\sigma$ | $\omega$-$\beta$-$\sigma$ | $\omega$-$\beta$-$\sigma$ | $\omega$-$\beta$-$\sigma$ | $\omega$-$\beta$-$\sigma$ |
| $R^2$ | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| Observations | 15,447,959 | 11,708,791 | 15,447,959 | 11,708,791 | 11,708,791 |
| Median fin. wealth | 256.2 | 176 | 256.2 | 176 | 176 |
| Median fin. wealth $\leq 600k$ NOK | | | 176 | | |
| Median fin. wealth $> 600k$ NOK | | | 1110.7 | | |

Notes: All specifications include age-group fixed effects. The flexible controls in all specifications include a four-way interaction between 8 bins of exposure to directly-held domestic stocks, 7 bins of betas for the observed part of the stock portfolio, 7 bins of volatility for the observed part of the stock portfolio and year. Specifications (2), (4), and (5) are restricted to gross financial wealth of up to 600k NOK in 2010 prices. An entrepreneur is defined as a household that owns more than 1/3 of the book value of stocks in an incorporated non-financial firm with at most 3 stock owners which employs at least one worker. For transition to entrepreneurship we only consider newly-created firms and households who have not owned stocks in any private firm in the past. In addition, we require that the newly-created firm does not own publicly traded domestic stocks in the year of foundation unless it has employees that are not members of the entrepreneur's household. Furthermore, upon transitioning to entrepreneurship a household is dropped from our sample. Buy-and-hold returns $r_{i,t}$ are defined as $r_{i,t} = r_t^f + \omega_{i,t-1}\mathbf{s}'_{i,t-1}\mathbf{r}_t$, where $r_t^f$ is the risk-free return in year $t$, $\omega_{i,t-1}$ denotes the share of financial wealth held in domestic stocks at the end of year $t-1$, $\mathbf{s}_{i,t-1}$ the weight in the stock portfolio of each domestic stock, and $\mathbf{r}_t$ the vector of realized excess returns of domestic stocks in year $t$. All coefficient estimates are scaled by 100 for easier interpretation. Standard errors in parentheses are clustered at the municipality level with a total of 422 clusters in each specification. + denotes significance at the 10% level, * denotes significance at the 5% level, and ** denotes significance at the 1% level.

## 5.2 Robustness

We perform a number of robustness exercises and specification tests. In all of our robustness exercises we focus on the specification in column (2) of Table 2 as the baseline for concreteness. Table 3 presents robustness to changing the set of covariates. Column (1) adds 9 bins of financial wealth interacted with year to absorb any correlation between *ex ante* wealth and the propensity to start a firm. Column (2) controls for lagged log labor earnings, since as shown in Section 2 labor market earnings serve as an opportunity cost in the business creation decision of a potential entrepreneur. Column (3) includes a richer set of portfolio controls based on the Fama and French (1993) three factor model: a 6-way interaction between 8 bins of exposure to directly-held domestic stocks, 7 bins of market betas from a Fama-French 3-factor model, 7 bins of volatility for the observed part of the portfolio, 2 bins for loading on the SMB (small minus big) factor, 2 bins for loading on the HML (high minus low) factor and year. Column (4) controls for the interaction between the NACE sector of a direct owner's largest portfolio holding and year to account for stock-owners loading on industries that they believe would do well and subsequently starting firms in those industries. The main regression coefficient changes little with any of these additional covariates. Column (5) considers a specification that controls for portfolio composition by including a three-way interaction of bins of the share of financial wealth invested in domestic stock mutual funds and ETFs, bins of the exposure to directly-held domestic stocks and year. In this way we account for possible systematic differences in portfolio composition and the propensity to enter into business.[23] In particular, this flexible control can help account for systematic differences in financial literacy, which lead to households holding undiversified portfolios of stocks versus more diversified mutual fund holdings. These controls turn out to matter little for our coefficient estimate. Finally, column (6) illustrates the importance of the flexible controls for *ex ante* portfolio heterogeneity by removing them altogether. The estimated coefficient is now substantially reduced and less significant. This points to *ex ante* portfolio heterogeneity being an important confounder for the link between stock returns and entrepreneurship.

[23] In our measure of portfolio composition we focus only on stock mutual fund and domestic stock ownership, since other holdings of asset classes, such as bond mutual funds, bonds, or international financial assets are much more concentrated in Norway – see Table A.5.

Table 3: Robustness to Additional Covariates

| | Dep. var.: becomes entrepreneur ($E_{i,t}$) | | | | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $r_{i,t}$ | 0.226** | 0.212** | 0.219** | 0.218** | 0.204* | 0.083* |
| | (0.073) | (0.077) | (0.082) | (0.074) | (0.085) | (0.036) |
| Location-year FE | Yes | Yes | Yes | Yes | Yes | Yes |
| Primary sector-year FE | Yes | Yes | Yes | Yes | Yes | Yes |
| Financial wealth-year FE | Yes | No | No | No | No | No |
| Portfolio-year FE | $\omega$-$\beta$-$\sigma$ | $\omega$-$\beta$-$\sigma$ | 3-factor model | $\omega$-$\beta$-$\sigma$ | $\omega$-$\beta$-$\sigma$ | None |
| $R^2$ | 0.002 | 0.002 | 0.003 | 0.002 | 0.002 | 0.002 |
| Observations | 11,708,791 | 11,004,463 | 11,643,550 | 11,708,553 | 8,455,315 | 11,803,707 |
| Description | Wealth bins | Labor income control | 3-factor model | Addn'l. sectoral controls | Portfolio composition | No portfolio characteristics |

Notes: All specifications include age-group fixed effects. Flexible controls in specifications (1), (2), (4) and (5) include a four-way interaction between 8 bins of exposure to directly-held domestic stocks, 7 bins of betas for the observed part of the portfolio, 7 bins of volatility for the observed part of the portfolio and year. Flexible controls in specification (3) include a 6-way interaction between 8 bins of exposure to directly-held domestic stocks, 7 bins of market betas from a Fama-French 3-factor model (Fama and French, 1993) for the observed part of the portfolio, 7 bins of volatility for the observed part of the portfolio, 2 bins for exposure to the SMB factor, 2 bins for exposure to the HML factor and year. Additional controls in specification (1) include 6 bins of financial wealth times year. Additional controls in specification (2) include lagged labor market income. Additional controls in specification (4) include an interaction between the level 1 NACE sector of the largest direct portfolio holding and year. Specification (5) includes a three-way interaction of 8 bins of share of financial wealth invested in directly-held domestic stocks, 5 bins of share of financial wealth held in domestic mutual funds and year. Note that for specification (5) we drop observations after 2015. We define an entrepreneur as an individual that owns more than 1/3 of the book value of stocks in an incorporated non-financial firm with at most 3 stock owners which employs at least one worker. When considering the transition to entrepreneurship we only consider newly-created firms and households who have not owned stocks in any private firm in the past. In addition we require that the newly-created firm does not own publicly traded domestic stocks in the year of foundation unless it has employees that are not members of the entrepreneur's household. Furthermore, upon transitioning to entrepreneurship a household is dropped from our sample. Buy-and-hold returns $r_{i,t}$ are defined as $r_{i,t} = r^f_t + \omega_{i,t-1}\mathbf{s}'_{i,t-1}\mathbf{r}_t$, where $r^f_t$ is the risk-free return in year $t$, $\omega_{i,t-1}$ denotes the share of financial wealth held in domestic stocks at the end of year $t-1$, $\mathbf{s}_{i,t-1}$ the weight in the stock portfolio of each domestic stock, and $\mathbf{r}_t$ the vector of realized excess returns of domestic stocks in year $t$. All specifications are restricted to gross financial wealth of up to 600k real NOK. All coefficient estimates are scaled by 100 for easier interpretation. Standard errors in parentheses are clustered at the municipality level with a total of 422 clusters in each specification. + denotes significance at the 10% level, * denotes significance at the 5% level, and ** denotes significance at the 1% level.

Table 4: Robustness to Alternative Return Definitions

| | Dep. var.: becomes entrepreneur ($E_{i,t}$) | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| $r_{i,t}$ | 0.242** | 0.381* | | |
| | (0.093) | (0.175) | | |
| $r_{i,t}^{noemp}$ | | | 0.252** | |
| | | | (0.074) | |
| $r_{i,t+1}$ | | | | 0.064 |
| | | | | (0.060) |
| Location-year FE | Yes | Yes | Yes | Yes |
| Primary sector-year FE | Yes | Yes | Yes | Yes |
| Financial wealth-year FE | No | No | | |
| Portfolio-year FE | $\omega$-$\beta$-$\sigma$ | $\omega$-$\beta$-$\sigma$ | $\omega$-$\beta$-$\sigma$ | $\omega$-$\beta$-$\sigma$ |
| $R^2$ | 0.002 | 0.002 | 0.002 | 0.002 |
| Observations | 11,521,512 | 11,296,110 | 11,708,791 | 11,655,703 |
| Description | < 3 stock owner | < 3 stock owner (top 20 stocks) | Own employer stock | Placebo returns |

Notes: All specifications include age-group fixed effects. Flexible controls in all specifications include a four-way interaction between 8 bins of exposure to directly-held domestic stocks, 7 bins of betas for the observed part of the portfolio, 7 bins of volatility for the observed part of the portfolio and year. We define an entrepreneur as an individual that owns more than 1/3 of the book value of stocks in an incorporated non-financial firm with at most 3 stock owners which employs at least one worker. When considering the transition to entrepreneurship we only consider newly-created firms and households who have not owned stocks in any private firm in the past. In addition we require that the newly-created firm does not own publicly traded domestic stocks in the year of foundation unless it has employees that are not members of the entrepreneur's household. Furthermore, upon transitioning to entrepreneurship a household is dropped from our sample. Buy-and-hold returns $r_{i,t}$ are defined as $r_{i,t} = r_t^f + \omega_{i,t-1}\mathbf{s}'_{i,t-1}\mathbf{r}_t$, where $r_t^f$ is the risk-free return in year $t$, $\omega_{i,t-1}$ denotes the share of financial wealth held in domestic stocks at the end of year $t-1$, $\mathbf{s}_{i,t-1}$ the weight in the stock portfolio of each domestic stock, and $\mathbf{r}_t$ the vector of realized excess returns of domestic stocks in year $t$. $r_{i,t}^{noemp}$ is the buy-and-hold portfolio return that replaces the return of the firm which the head of the household is employed in with the OBX return for that year. The placebo return $r_{i,t+1} = r_{t+1}^f + \omega_{i,t-1}\mathbf{s}'_{i,t-1}\mathbf{r}_{t+1}$, where $r_{t+1}^f$ is the risk-free return in year $t+1$, $\omega_{i,t-1}$ denotes the share of financial wealth held in domestic stocks at the end of year $t-1$, $\mathbf{s}_{i,t-1}$ the weight in the stock portfolio of each domestic stock, and $\mathbf{r}_{t+1}$ the vector of realized excess returns of domestic stocks in year $t+1$. All specifications are restricted to gross financial wealth of up to 600k real NOK. All coefficient estimates are scaled by 100 for easier interpretation. Standard errors in parentheses are clustered at the municipality level with a total of 422 clusters in each specification. + denotes significance at the 10% level, * denotes significance at the 5% level, and ** denotes significance at the 1% level.

Table 4 explores changes to the return definition or restricting variation in returns. Column (1) narrows the stock portfolio variation to just households that hold directly less than 3 stocks. The coefficient estimate is largely unchanged, reflecting the fact that most of the variation in domestic stock portfolios in the data comes from owners of less than 3 stocks (see Table A.2). Column (2) further requires the main stock holding of such households to be among the 20 most popular companies traded by small investors on the Norwegian stock exchange, to help rule out undiversified investments in "exotic" single stocks due to private information or superior stock picking skill correlated with the propensity to start a firm.[24] In column (3) we account for stock owners holding stocks in their employer by replacing the firm-specific return for the firm where the household head is employed with the OBX return. This accounts for positive employer-specific returns due to innovative activity by the employer that may in turn trigger an idea spillover and spur additional business creation by employees (Babina and Howell, 2022; Chetty et al., in progress). Finally, column (4) performs a placebo exercise using the portfolio return in year $t+1$. The estimated coefficient is close to zero and insignificant in that case, bolstering the causal interpretation of our main effect.

Table A.7 in the Appendix reports results from additional robustness exercises. First, rather than computing the market beta using the domestic OBX index, which may be tilted towards energy stocks, we use the CRSP value-weighted index. Second, we consider the effects of trimming our sample for extreme realizations of exposure and stock portfolio characteristics. The estimates are slightly lower but still highly significant. Third, we restrict the sample only to households that do not receive business income in year $t-1$ to rule out possible changes in legal form of unincorporated businesses as opposed to new business creation. Fourth, in column (5) we modify the entrepreneurship definition by also considering existing firms that start hiring employees in addition to newly-created firms. We find positive and statistically significant effects of returns on entrepreneurship for this alternative definition as well. Fifth, we find a slightly smaller but still statistically significant response in a specification with buy-and-hold kroner gains or losses (in thousands of 2010 NOK) rather than portfolio returns, consistent with the marginal effect decreasing in wealth. Finally, to account for stock wealth held in non-taxable retirement accounts in the measure of gross financial wealth, we replace in the denominator of $r$ the value of stock wealth held in Norwegian public stocks on tax returns with the value of the domestic stock portfolio from the stock register, with little change.

# 6 Results on Firm-level Outcomes

We now investigate how the entrepreneur's stock market wealth affects the characteristics of the new firm, using Proposition 3. In addition to being of interest in their own right, these results help to distinguish financial frictions from other explanations for the effect on firm entry.

[24] See Table A.6 in the Appendix for a list of these stocks.

## 6.1 Implementation Details

We implement the selection correction as follows. We restrict attention to non-negative stock returns and extend the exposition in Section 2 to incorporate $M = 4$ return bins: [0%,10%], (10%, 20%], (20%, 50%], and over 50%. Denote these return bins by $m \in \{1, 2, 3, 4\}$. For each bin below the highest, $m \in \{1, 2, 3\}$, denote by $p_m(x_i)$ the probability that the return is in the highest return bin rather than bin $m$, conditional on the covariates $x_i$. We estimate $p_m(x_i)$ using the sample of stock owners in bin $m$ and in bin 4, controlling for the covariates from our baseline specification.[25] We obtain the propensity score weight $\omega(x_i)$ for each observation by applying the formula in Eq. (2), noting that $dF_a\left(a^H|x_i\right)/dF_a\left(a^m|x_i\right) = p_m(x_i)/(1 - p_m(x_i))$ and $dF_a\left(a^m\right)/dF_a\left(a^H\right)$ equals the relative share of the population in each bin.[26]

With the propensity score weights in hand, we calculate the reweighted probability $e^*(a^m, k \geq \bar{k})$ of becoming an entrepreneur for each return bin. Following Proposition 3, we then truncate the reweighted-distribution of initial assets of newly-started firms in return bin $m$ at the $(1 - e_1^*/e_m^*) \times 100$-th percentile. This step is the adjustment for selection into entrepreneurship. Finally, we estimate the effect of a higher portfolio return in the truncated sample via a weighted regression that uses the propensity score weights of firm income and balance-sheet statement and household financial outcomes on the average portfolio return in the entrepreneur's return bin. We report bootstrap standard errors that account for the estimated propensity score weights.[27]

## 6.2 Baseline Results

Table 5 reports the results for firm income statement (top panel), balance sheet (middle panel) and household-level (bottom panel) outcomes in the year of foundation. The income and balance sheet items (except employment) are in thousands of 2010 NOK and annualized to adjust for differences in foundation dates. The household outcomes (apart from the pre-entry log earnings) are scaled by $t - 1$ gross financial wealth and multiplied by 100.

Higher wealth implies sizable positive effects for sales, employment, the wage bill, and value added. In terms of magnitudes, a 20 percentage point higher return increases these

[25] Due to computational costs associated with using a non-linear model on our large population and given the multiple fixed effects, we estimate these propensities using a linear probability model. Since the linear probability model may give propensities close to or above unity, which implies an undefined or very large value for the propensity score weight, we drop observations with estimated propensities above 90%.

[26] We additionally force the propensity score weights to average to unity, as advocated by Busso et al. (2009) to improve the performance of the propensity score reweighting procedure.

[27] Specifically, for each bootstrap iteration, we generate weights from a $\Gamma(1, 1)$ distribution, fixing the weight over time for each household. We then estimate propensity score weights on the bootstrap-weighted sample and implement the selection correction and firm or entrepreneur-level regression using the estimated propensity score weights and store the regression coefficients.

Table 5: Firm and Entrepreneur Outcomes

| | Sales | Wage bill | Empl. | Value Added | VA/worker | EBITDA |
|---|---|---|---|---|---|---|
| $r_{i,t}$ | $54.7^{**}$ | $11.1^{*}$ | $0.04^{*}$ | $24.7^{*}$ | $-2.6$ | $7.1^{**}$ |
| | (19.5) | (5.4) | (0.02) | (11.3) | (4.2) | (2.7) |
| N | 736 | 736 | 736 | 736 | 736 | 736 |
| Mean | 1832.8 | 662.9 | 2.2 | 1246.2 | 739.8 | 236.6 |
| Median | 1383 | 555.8 | 1 | 927.1 | 495.3 | 150.1 |

| | Tot. assets | Fixed assets | Wk. Cap. | Equity | Tot. Liab. |
|---|---|---|---|---|---|
| $r_{i,t}$ | $21.2^{**}$ | $2.6^{*}$ | $3.5^{+}$ | $2.9^{*}$ | $17.6^{**}$ |
| | (6.9) | (1.2) | (1.9) | (1.3) | (5.6) |
| N | 736 | 736 | 736 | 736 | 736 |
| Mean | 797.5 | 119.8 | 176.9 | 200.4 | 546 |
| Median | 458 | 33.6 | 108.5 | 139.2 | 318.8 |

| | Private firm equity | Change in stock holdings | Change in h.h. debt | Log of pre-entry earnings |
|---|---|---|---|---|
| $r_{i,t}$ | $1.10^{**}$ | $-0.23^{*}$ | 1.38 | $-0.003$ |
| | (0.35) | (0.09) | (1.28) | (0.01) |
| N | 736 | 736 | 736 | 692 |
| Mean | 40.92 | -1.96 | 28.4 | 13.07 |
| Median | 21.84 | 0 | -8.93 | 13.23 |

Notes: All monetary values are in thousands of 2010 NOK. Outcomes are winsorized at the 5th and 95th percentiles. Working capital is defined as the difference between current assets and current liabilities. Entrepreneur balance sheet outcomes in the bottom panel are relative to lagged gross financial wealth. "Private firm equity" denotes the book value of a household's holdings of private firm equity relative to lagged gross financial wealth. "Change in stock holdings" is the change in the value of the portfolio of directly held stocks given constant stock prices between $t-1$ and $t$ relative to lagged gross financial wealth. "Change in h.h. debt" is the change in total household debt over lagged gross financial wealth. "Log of pre-entry earnings" is the log of previous year's labor market earnings. The first three outcomes in the bottom table are scaled by 100 for easier interpretation. The results are based on the selection correction procedure described in Section 6. Bootstrapped standard errors in parentheses. + denotes significance at the 10% level, * denotes significance at the 5% level, and ** denotes significance at the 1% level.

variables by between 30 and 60% of the mean. Crucially, higher wealth also increases total earnings (EBITDA), consistent with a financial frictions channel but not a non-pecuniary benefit channel. In contrast, and consistent with our theoretical framework underpinning the selection correction procedure, there is a much smaller and insignificant effect on firm earnings without implementing the selection correction – see Table A.8 in the Appendix.

Turning to the balance sheet, total assets and fixed (tangible and intangible) assets also increase. The increase in owners' equity in the firm of 2.9k NOK for a 1% higher stock return implies a nearly one-to-one pass-through of stock wealth to owners' equity; specifically, a 1p.p. higher return on median financial wealth of 256k NOK amounts to 2.56k NOK higher stock wealth, almost exactly equal to the regression coefficient of 2.9.

The third panel provides evidence on how households finance the marginal increase in the size of their firm. The first three columns in this panel scale the dependent variable by lagged financial wealth, the same denominator as used to construct $r_{i,t}$, so that the coefficients have the interpretation of the marginal NOK change per additional NOK of stock wealth. Thus, household equity in the firm rises by 1.1 NOK for each additional 1 NOK of stock wealth and the data do not reject a pass-through of one. This pass-through mirrors the near one-to-one increase in total owners' equity in the firm in response to higher entrepreneur starting wealth. There is no mechanical reason that the estimated impact on owners' equity in the firm need coincide with the estimated impact on the household's holding of private firm equity, nor that either pass-through should lie near one. The fact that they do lends some credence to the selection correction procedure and bolsters the link between higher stock wealth and firm outcomes.

Households may fund their increase in private firm equity by liquidating publicly-traded stocks, borrowing, or using other savings. On average, households liquidate around 23 cents of their initial public equity position for every additional NOK of equity in the firm.[28] There is also a positive but statistically insignificant increase in household borrowing. The near one-to-one pass-through of marginal stock market wealth into firm equity and the evidence that households liquidate part of their portfolio and borrow to finance a larger firm further suggest the importance of liquidity constraints as a key friction making stock wealth relevant for business creation.

## 6.3 Robustness

Proposition 3 requires that the entry and asset choice decisions depend only on wealth $a$ and business productivity $z$. Section 2.5 highlighted the robustness of these results to *ex ante* and *ex post* residual heterogeneity in size or profits which is independent from wealth $a$ and business productivity $z$ up to a potential bias close to the entry cutoff. In that case, looking at the subsample of firms which are sufficiently away from the entry cutoff mitigates the potential bias. Accordingly, Table A.9 in the Appendix presents a robustness exercise that accounts for residual heterogeneity by dropping the bottom 10% of firms in each return

[28] We also compare the average stock portfolio liquidation of entrepreneurs relative to non-entrepreneurs that are ex ante identical and who end up in the the same ex post return bin. For stock owners with portfolio return higher than 10%, entrepreneurs liquidate on average around 6% more of their stock holdings as a share of lagged financial wealth, compared to non-entrepreneurs.

bin by size after the selection correction procedure. The coefficients change little, while the standard errors increase due to the reduced sample size. Therefore, we conclude that our firm level outcome results appear robust to possible biases due to residual heterogeneity.

Finally, we include robustness exercises that address specific sources of residual heterogeneity. One concern is that individuals might vary in their outside option of labor income relative to business productivity. This heterogeneity can imply that some individuals with high labor income and relatively lower productivity enter only if they have higher initial wealth. This violation has a testable implication, since if the conditions in our model hold, then the distributions of $z$ and the wage if work $w(z)$ should not vary with wealth after applying Proposition 3. Accordingly, the final regression reported in Table 5 has the pre-entry wage as the dependent variable. Consistent with the model's assumptions, we find no evidence of a difference in pre-entry wage by wealth.

A related concern is that individuals might have industry-specific skills that imply variation in their prospective firms' production processes or startup costs. Table A.3 reports the variation in median assets in the year of foundation by NACE sector. While most sectors have broadly similar initial sizes with assets in the range of 400-800k NOK, a few have much larger typical sizes, with utilities the largest at more than 14,000k NOK. Interestingly, Table A.3 also shows that sectors with very high typical initial size account for a relatively small share of new firms. This suggests that differences in capital intensity likely do not matter much for our firm-level outcomes. Table A.10 confirms this intuition by showing that the firm-level results are similar to and if anything for the most part slightly larger than our baseline when removing firms in sectors with high or very low start-up size.[29] The data do not reject equality of any coefficient reported in Table 5 with its counterpart in Table A.10.

# 7 Conclusion

In this paper we provide evidence that more stock market wealth causally increases business creation. The effects concentrate among moderate financial wealth individuals and in years when aggregate stock returns are high. This confluence points to a special role for stock market wealth in that stock wealth increases precisely when the returns to entrepreneurship are high.

Determining the effect of wealth on firm outcomes requires accounting for selection into entrepreneurship. Applying our model-motivated selection correction, we find that wealthier

[29] The high start-up size sectors are Electricity, gas, steam, and air conditioning supply, Water supply, Mining and quarrying, Financial and insurance activities, Agriculture, forestry and fishing, and Real estate activities. The two sectors with low start-up sizes are Other service activities and Administrative and support service activities.

entrepreneurs start larger, more profitable firms. Together with the absence of marginal effects in very high wealth households and the near one-to-one pass-through of marginal stock market wealth into firm equity, the positive effect on profits signifies financial frictions as a key mechanism for why wealth affects entrepreneurship.

Finally, our firm-level findings illustrate the importance of household wealth for business creation and growth. Initial firm size is a key determinant of long-run firm size and performance (Sedláček and Sterk (2017), Sterk et al. (2021)), raising the possibility of long-run effects of the stock market on economic growth via a business creation channel.

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Stock Market Wealth and Entrepreneurship

Online Appendix

Gabriel Chodorow-Reich  Plamen Nenov  Vitor Santos  Alp Simsek

# A Data Appendix

Table A.1: Summary Statistics of New Firms (thousands of 2010 NOK)

| | **All years** | | **Low return years** | | **High return years** | |
|---|---|---|---|---|---|---|
| | Mean | Median | Mean | Median | Mean | Median |
| Total Assets | 928.0 | 478.8 | 861.0 | 454.9 | 996.9 | 503.6 |
| Sales | 1851.5 | 819.0 | 1714.2 | 789.3 | 1992.9 | 848.7 |
| Wage Bill | 542.1 | 316.0 | 525.8 | 306.2 | 558.9 | 323.9 |
| Employment | 3.3 | 2.0 | 3.1 | 2.0 | 3.5 | 2.0 |
| Value added | 985.9 | 579.9 | 962.5 | 567.3 | 1010.0 | 593.0 |
| Value added / worker | 417.8 | 276.8 | 428.0 | 279.0 | 407.4 | 275.1 |
| Working Capital | 120.8 | 66.7 | 112.7 | 68.0 | 129.2 | 64.9 |
| Fixed Assets | 273.0 | 48.0 | 251.3 | 44.3 | 295.4 | 51.1 |
| Tangibles | 209.3 | 26.3 | 194.1 | 23.5 | 225.0 | 29.5 |
| Equity | 186.8 | 104.7 | 181.8 | 102.0 | 192.0 | 107.2 |

Table A.2: Distribution of Portfolio Characteristics for Stock Holders

| Percentile | $\beta$ | $\omega$ | $\sigma$ | **Herfindahl index** | **Number of stocks** | **Change in stock holdings (in %)** | **Share stock mutual funds** | **Share deposits** |
|---|---|---|---|---|---|---|---|---|
| 10th | 0.45 | 0.02 | 0.01 | 0.34 | 1 | -100.00 | 0.00 | 0.16 |
| 20th | 0.63 | 0.03 | 0.01 | 0.47 | 1 | -18.01 | 0.00 | 0.29 |
| 30th | 0.76 | 0.05 | 0.01 | 0.56 | 1 | -0.00 | 0.00 | 0.40 |
| 40th | 0.87 | 0.07 | 0.02 | 0.76 | 1 | 0.00 | 0.00 | 0.50 |
| 50th | 0.97 | 0.10 | 0.02 | 0.99 | 2 | 0.00 | 0.02 | 0.60 |
| 60th | 1.00 | 0.14 | 0.02 | 1.00 | 2 | 0.00 | 0.06 | 0.69 |
| 70th | 1.04 | 0.20 | 0.02 | 1.00 | 3 | 4.67 | 0.10 | 0.77 |
| 80th | 1.16 | 0.29 | 0.03 | 1.00 | 3 | 25.37 | 0.17 | 0.85 |
| 90th | 1.32 | 0.44 | 0.03 | 1.00 | 5 | 76.78 | 0.30 | 0.92 |

Notes: $\omega$ is the share of directly held domestic stocks out of total financial wealth. "Share stock mutual funds" is the portfolio share invested in domestic stock mutual funds and exchange-traded funds. "Share deposits" is the share of financial wealth held in deposits. Note that the stock mutual fund shares are for the period 2004-2015. "Change in stock holdings" is the change in the value of the portfolio of directly held stocks given constant stock prices between $t-1$ and $t$ relative to lagged gross financial wealth.

Table A.3: Industry Distribution of New Firms, 2009-2019.

| Industry | Share (%) | Median total assets (1000 NOK) |
|---|---|---|
| Construction | 23.10 | 467 |
| Wholesale and retail trade and repair of motor vehicles | 20.57 | 774 |
| Professional, scientific and technical activities | 16.08 | 398 |
| Administrative and support service activities | 6.71 | 376 |
| Information and communication | 5.93 | 396 |
| Human health and social work activities | 5.66 | 473 |
| Manufacturing | 4.49 | 757 |
| Transportation and storage | 4.38 | 661 |
| Other service activities | 3.59 | 293 |
| Real estate activities | 3.05 | 1297 |
| Education | 1.9 | 278 |
| Arts, entertainment and recreation | 1.81 | 394 |
| Agriculture, forestry and fishing | 1.35 | 1074 |
| Financial and insurance activities | 0.72 | 3572 |
| Mining and quarrying | 0.27 | 870 |
| Water supply, sewerage, waste management | 0.22 | 1380 |
| Electricity, gas, steam and air conditioning supply | 0.16 | 14133 |

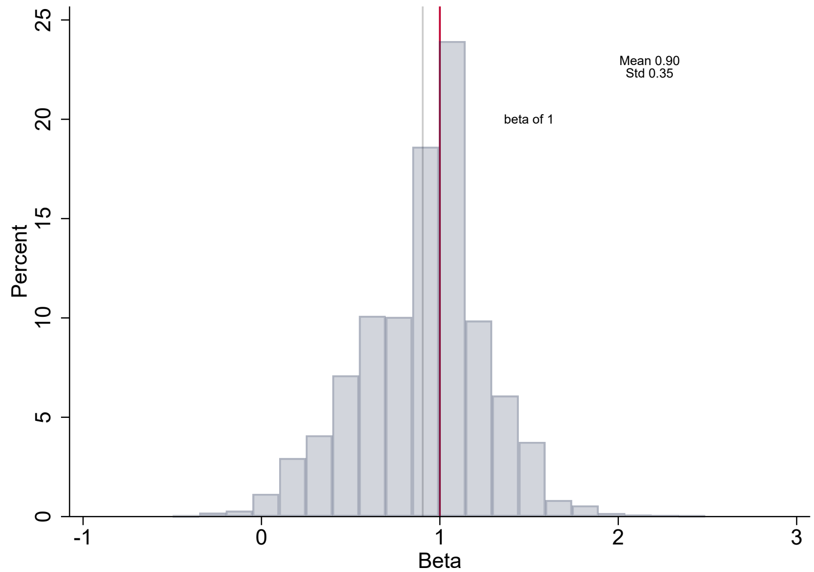
Notes: Monetary amounts are in 1000s of 2010 Norwegian kroner.

Table A.4: Sample Descriptive Statistics.

| | **Non-Entrepreneur** | | | | | **Entrepreneur** | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **Mean** | **p25** | **p50** | **p75** | **Share** | **Mean** | **p25** | **p50** | **p75** | **Share** |
| Age | 44.7 | 35 | 45 | 55 | . | 39.8 | 32 | 39 | 47 | . |
| Earnings | 648.7 | 339.2 | 591.9 | 912.9 | . | 655.3 | 362.7 | 608.1 | 875.3 | . |
| Financial Wealth | 579.1 | 120.0 | 269.7 | 639.9 | . | 528.1 | 143.2 | 291.5 | 606.2 | . |
| Directly held stocks | 143.2 | 13.4 | 46.6 | 137.8 | . | 108.9 | 6.4 | 29.3 | 98.3 | . |
| **Share** | – | – | – | – | **99.79** | – | – | – | – | **.21** |
| | **(Direct) Stock owner** | | | | | **Other** | | | | |
| | **Mean** | **p25** | **p50** | **p75** | **Mean** | **p25** | **p50** | **p75** | **Share** | |
| Age | 47 | 38 | 48 | 57 | . | 44.3 | 34 | 45 | 55 | . |
| Earnings | 884.9 | 495.5 | 836.2 | 1176.4 | . | 614.8 | 322.1 | 560.7 | 874.7 | . |
| Financial Wealth | 1031.2 | 276.3 | 601.1 | 1298.0 | . | 514.0 | 110.5 | 241.3 | 557.9 | . |
| Directly held stocks | 154.5 | 18.6 | 54.4 | 151.2 | . | 0.0 | 0.0 | 0.0 | 0.0 | . |
| **Share** | – | – | – | – | **12.56** | – | – | – | – | **87.44** |
| | **Moderate Wealth (<600K)** | | | | | **High Wealth (>600K)** | | | | |
| | **Mean** | **p25** | **p50** | **p75** | **Share** | **Mean** | **p25** | **p50** | **p75** | **Share** |
| Age | 43 | 33 | 43 | 53 | . | 49.6 | 42 | 51 | 59 | . |
| Earnings | 607.1 | 328.6 | 561.2 | 864.8 | . | 774.8 | 377.4 | 709.5 | 1079.4 | . |
| Financial Wealth | 270.6 | 96.0 | 187.1 | 345.7 | . | 1514.6 | 745.0 | 1117.6 | 1857.2 | . |
| Directly held stock | 59.2 | 7.3 | 24.9 | 69.3 | . | 244.4 | 34.1 | 98.2 | 271.3 | . |
| **Share** | – | – | – | – | **75.21** | – | – | – | – | **24.79** |

Notes: Monetary amounts are in 1000s of 2010 Norwegian kroner.

Figure A.1: Beta of Portfolio of Directly Held Domestic Stocks

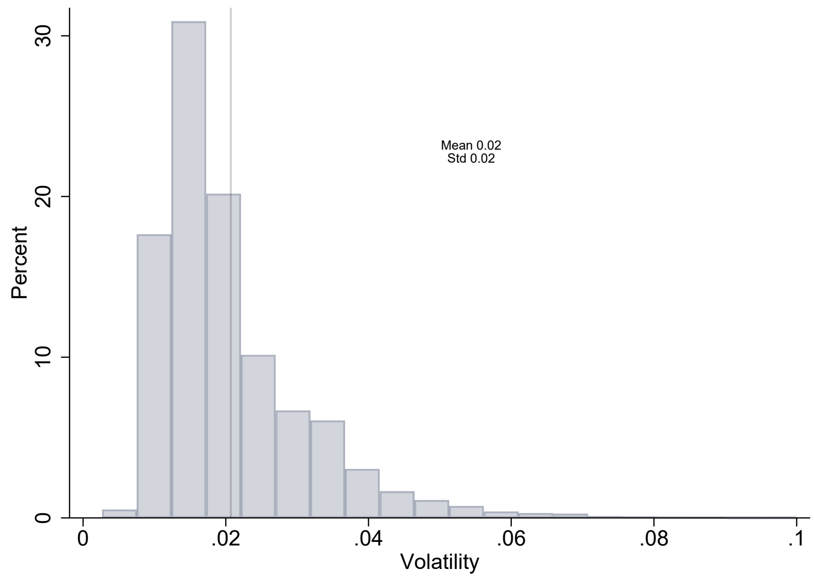



Notes: We measure the portfolio characteristics $\beta_{i,t-1}$ and $\sigma_{i,t-1}$ using daily returns over the year $t-1$. Specifically, for each observation we form the time series of daily returns $\tilde{r}_{i,t-1+\Delta} = \omega_{i,t-1}^{-1}\mathbf{s}_{i,t-1}\mathbf{r}_{t-1+\Delta}$, where $\tilde{r}_{i,t-1+\Delta}$ gives the return on day $t-1+\Delta$ of a portfolio with weights fixed at their value at the end of the year. We obtain $\beta_{i,t-1}$ as the OLS regression coefficient from a regression of $\tilde{r}_{i,t-1+\Delta}$ on $r_{t-1+\Delta}^{m}$ less the risk-free rate and $\sigma_{i,t}$ as the variance of $\tilde{r}_{i,t-1+\Delta}$.

Table A.5: Shares of the Population by Asset Holdings (2004-2015).

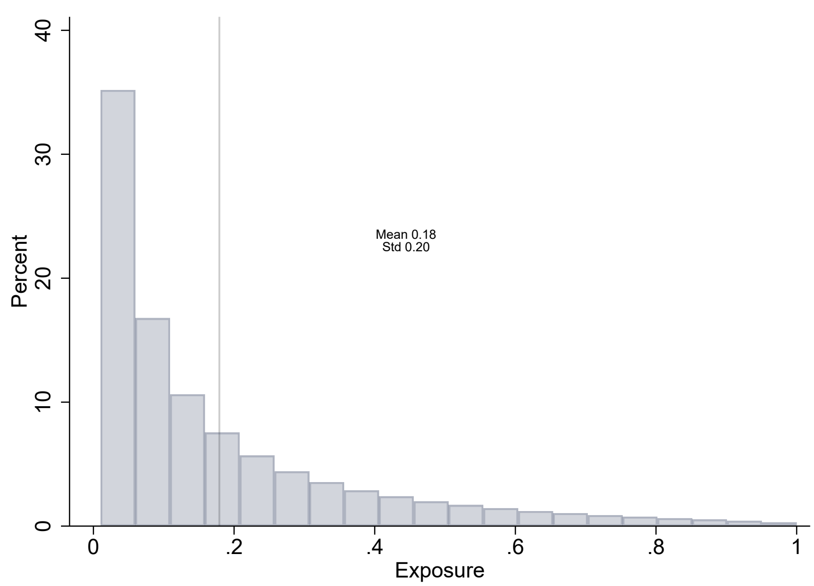
| | Population | Moderate Wealth ($\leq 600K$) | High wealth ($> 600K$) |
|---|---|---|---|
| 1. Direct owner domestic stock | 0.131 | 0.095 | 0.250 |
| Of which: $<3$ stocks | 0.085 | 0.070 | 0.134 |
| Of which: among top 20 | 0.055 | 0.044 | 0.093 |
| 2. Domestic stock mutual fund owner | 0.409 | 0.366 | 0.550 |
| 3. Direct or mutual fund owner | 0.459 | 0.409 | 0.621 |
| 4. Direct and mutual fund owner | 0.082 | 0.051 | 0.180 |
| Of which: $<3$ stocks | 0.050 | 0.037 | 0.093 |
| Of which: among top 20 | 0.033 | 0.023 | 0.066 |
| 5. Domestic bond mutual fund owner | 0.092 | 0.079 | 0.131 |
| 6. Domestic bond owner | 0.012 | 0.006 | 0.032 |
| 7. Foreign assets owner | 0.055 | 0.036 | 0.117 |

Figure A.2: Volatility of Portfolio of Directly Held Domestic Stocks




Notes: We measure the portfolio characteristics $\beta_{i,t-1}$ and $\sigma_{i,t-1}$ using daily returns over the year $t-1$. Specifically, for each observation we form the time series of daily returns $\tilde{r}_{i,t-1+\Delta} = \omega_{i,t-1}^{-1} \mathbf{s}_{i,t-1} \mathbf{r}_{t-1+\Delta}$, where $\tilde{r}_{i,t-1+\Delta}$ gives the return on day $t-1+\Delta$ of a portfolio with weights fixed at their value at the end of the year. We obtain $\beta_{i,t-1}$ as the OLS regression coefficient from a regression of $\tilde{r}_{i,t-1+\Delta}$ on $r^m_{t-1+\Delta}$ less the risk-free rate and $\sigma_{i,t}$ as the variance of $\tilde{r}_{i,t-1+\Delta}$.

Figure A.3: Exposure to Domestic Stocks




Notes: $\omega_{i,t}$ denotes the share of financial wealth held in domestic stocks at the end of year $t$.

Table A.6: List of the 20 Most Popular Publicly Traded Companies Held by Direct Owners of Less Than 3 Domestic Stocks

| **5-digit NACE sector** | **Average Mkt. Cap.** | **Average Number** | **Fraction shareholders** | **Ownership share** | **Share wealth invested** |
|---|---|---|---|---|---|
| Extraction of crude petroleum | 1083.2 | 36.8 | 0.35 | 0.01 | 0.21 |
| Wireless telecom. activities | 1053.0 | 18.9 | 0.35 | 0.01 | 0.07 |
| Manufact. prepared meals and dishes | 143.0 | 15.8 | 0.32 | 0.02 | 0.10 |
| Non-life insurance | 109.4 | 13.9 | 0.31 | 0.02 | 0.06 |
| Activities financial holding companies | 89.8 | 12.5 | 0.38 | 0.01 | 0.01 |
| Manufacturing paper and paperboard | 63.6 | 10.8 | 0.38 | 0.05 | 0.01 |
| Production primary aluminium | 547.7 | 9.0 | 0.15 | 0.01 | 0.04 |
| Other technical consultancy | 24.4 | 7.1 | 0.19 | 0.04 | 0.02 |
| Construction oil-platforms and modules | 554.9 | 5.2 | 0.28 | 0.01 | 0.01 |
| Manufacture fertilisers & nitrogen compounds | 120.0 | 5.2 | 0.11 | 0.003 | 0.01 |
| Operation marine fish farms | 152.6 | 5.0 | 0.18 | 0.01 | 0.02 |
| Extraction of crude petroleum | 3.6 | 4.5 | 0.20 | 0.05 | 0.03 |
| Scheduled long-dist. bus transports | 2.5 | 3.9 | 0.59 | 0.09 | 0.004 |
| Scheduled long-dist. transport coastal waters | 4.3 | 3.6 | 0.35 | 0.04 | 0.004 |
| Construction residential & non-residential buildings | 6.2 | 3.4 | 0.38 | 0.10 | 0.05 |
| Other monetary intermediation | 268.9 | 3.2 | 0.22 | 0.07 | 0.04 |
| Electricity production through water power | 14.5 | 3.1 | 0.28 | 0.01 | 0.01 |
| Wholesale computers computer equip. and software | 37.1 | 3.1 | 0.25 | 0.02 | 0.004 |
| Freezing of fish, fish fillets crustaceans and molluscs | 0.04 | 3.0 | 0.20 | 0.02 | 0.0004 |
| Other monetary intermediation | 172.8 | 2.5 | 0.37 | 0.14 | 0.02 |

Notes: "Average Mkt. Cap" is the average market capitalization of the company during our sample period (in billions of NOK). "Average number" is the average number of owners of less than 3 stocks (in thousands). "Fraction shareholders" is the share out of all stockholders in the company who are owners of less than 3 stocks. "Ownership share" is the share of the firm owned by owners of less than 3 stocks. "Share wealth invested" is the average share of the total stock market wealth of owners of less than 3 stocks that is invested in that particular company.

Table A.7: Additional Robustness

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| $r_{i,t}$ | 0.237** | 0.145** | 0.138* | 0.248* | | |
| | (0.078) | (0.049) | (0.065) | (0.116) | | |
| pot. gain | | | | | 0.063** | |
| | | | | | (0.020) | |
| $r_{i,t}^{alt}$ | | | | | | 0.203** |
| | | | | | | (0.066) |
| Loc.-year FE | Yes | Yes | Yes | Yes | Yes | Yes |
| Sector-year FE | Yes | Yes | Yes | Yes | Yes | Yes |
| Portfolio-year FE | $\omega$-$\beta_{CRSP}$-$\sigma$ | $\omega$-$\beta$-$\sigma$ | $\omega$-$\beta$-$\sigma$ | $\omega$-$\beta$-$\sigma$ | $\omega$-$\beta$-$\sigma$ | $\omega$-$\beta$-$\sigma$ |
| $R^2$ | 0.002 | 0.002 | 0.002 | 0.003 | 0.002 | 0.002 |
| Observations | 11,645,434 | 11,803,582 | 9,994,039 | 11,451,195 | 11,70,791 | 11,727,986 |
| Description | CRSP index | No trimming | No business income | $E_{i,t}$ or starting employment | Pot. change in wealth | Alternative fin. wealth |

Notes: All specifications include age-group fixed effects. Flexible controls include a four-way interaction between 8 bins of exposure to directly-held domestic stocks, 7 bins of betas for the observed part of the portfolio, 7 bins of volatility for the observed part of the portfolio and year. For specification (1) the betas are replaced with betas with respect to the CRSP value-weighted index. For specification (3) we restrict the sample to households that have not received business income in the year prior to the firm's foundation. For specification (5) we consider either our baseline entrepreneurship definition or existing firms transitioning to positive employment. We define an entrepreneur as an individual that owns more than 1/3 of the book value of stocks in an incorporated non-financial firm with at most 3 stock owners which employs at least one worker. When considering the transition to entrepreneurship we only consider newly-created firms and households who have not owned stocks in any private firm in the past. In addition we require that the newly-created firm does not own publicly traded domestic stocks in the year of foundation unless it has employees that are not members of the entrepreneur's household. Furthermore, upon transitioning to entrepreneurship a household is dropped from our sample. Buy-and-hold returns $r_{i,t}$ are defined as $r_{i,t} = r_t^f + \omega_{i,t-1}\mathbf{s}'_{i,t-1}\mathbf{r}_t$, where $r_t^f$ is the risk-free return in year $t$, $\omega_{i,t-1}$ denotes the share of financial wealth held in domestic stocks at the end of year $t-1$, $\mathbf{s}_{i,t-1}$ the weight in the stock portfolio of each domestic stock, and $\mathbf{r}_t$ the vector of realized excess returns of domestic stocks in year $t$. Kroner return$_{i,t}$ denotes the buy-and-hold gain/loss in thousands of 2010 NOK. $r_{i,t}^{alt}$ denotes an alternative buy-and-hold portfolio return formed by constructing domestic stock exposure using an alternative measure of gross financial wealth formed by replacing gross financial wealth with an alternative measure that replaces the value of stock wealth held in Norwegian public stocks on the tax returns with the value of the domestic stock portfolio we compute directly. All specifications are restricted to gross financial wealth of up to 600k real NOK. All coefficient estimates are scaled by 100 for easier interpretation. Standard errors in parentheses are clustered at the municipality level with a total of 422 clusters in each specification. + denotes significance at the 10% level, * denotes significance at the 5% level, and ** denotes significance at the 1% level.

Table A.8: Firm and Entrepreneur Outcomes – No Selection Correction

| | Sales | Wage bill | Empl. | Value Added | VA/worker | EBITDA |
|---|---|---|---|---|---|---|
| $r_{i,t}$ | 22.9* | 5.5+ | 0.02+ | 9.1 | −3.9 | 2.3 |
| | (10.8) | (2.9) | (0.01) | (6.4) | (3.5) | (1.8) |
| N | 797 | 797 | 797 | 797 | 797 | 797 |
| Mean | 1684.7 | 664.8 | 2.2 | 1146.2 | 682.1 | 202.5 |
| Median | 1330.1 | 591.1 | 1 | 909.8 | 427.9 | 97.1 |

| | Tot. assets | Fixed assets | Wk. Cap. | Equity | Tot. Liab. |
|---|---|---|---|---|---|
| $r_{i,t}$ | 9.1* | 1.3+ | 1.2 | 1.3 | 7.8* |
| | (4.1) | (0.8) | (1.4) | (0.9) | (3.2) |
| N | 797 | 797 | 797 | 797 | 797 |
| Mean | 748.6 | 116.4 | 161.3 | 189.9 | 514 |
| Median | 458 | 42.5 | 60.2 | 139.2 | 318.8 |

| | Private firm equity | Change in stock holdings | Change in h.h. debt | Log of pre-entry earnings |
|---|---|---|---|---|
| $r_{i,t}$ | 0.63** | −0.15+ | 1.23 | 0.002 |
| | (0.22) | (0.08) | (0.92) | (0.005) |
| N | 797 | 797 | 797 | 749 |
| Mean | 38.41 | -2.84 | 35.77 | 13.13 |
| Median | 21.84 | 0 | -1.84 | 13.31 |

Notes: The table corresponds to Table 5 without the selection correction procedure described in Section 6. All monetary values are in thousands of 2010 NOK. Outcomes are winsorized at the 5th and 95th percentiles. Working capital is defined as the difference between current assets and current liabilities. Entrepreneur balance sheet outcomes in the bottom panel are relative to lagged gross financial wealth. "Private firm equity" denotes the book value of a household's holdings of private firm equity relative to lagged gross financial wealth. "Change in stock holdings" is the change in the value of the portfolio of directly held stocks given constant stock prices between $t-1$ and $t$ relative to lagged gross financial wealth. "Change in h.h. debt" is the change in total household debt over lagged gross financial wealth. The first three outcomes in the bottom table are scaled by 100 for easier interpretation. Bootstrapped standard errors in parentheses. The rows labeled P(=Table 5) report the p-values from a bootstrapped t-test of equality between the coefficients in this table and in Table 5. + denotes significance at the 10% level, * denotes significance at the 5% level, and ** denotes significance at the 1% level.

Table A.9: Firm and Entrepreneur Outcomes – 10% trim

| | Sales | Wage bill | Empl. | Value Added | VA/worker | EBITDA |
|---|---|---|---|---|---|---|
| $r_{i,t}$ | 56.1$^{**}$ | 11.1$^{+}$ | 0.05$^{**}$ | 23.2$^{+}$ | $-6.1$ | 6.6$^{*}$ |
| | (20.4) | (5.8) | (0.02) | (11.9) | (4.4) | (2.9) |
| N | 662 | 662 | 662 | 662 | 662 | 662 |
| Mean | 2010 | 718.3 | 2.2 | 1375.6 | 808.9 | 265.8 |
| Median | 1502.6 | 607.3 | 1 | 1090.8 | 633.9 | 150.1 |

| | Tot. assets | Fixed assets | Wk. Cap. | Equity | Tot. Liab. |
|---|---|---|---|---|---|
| $r_{i,t}$ | 21.5$^{**}$ | 2.7$^{*}$ | 3.1 | 2.5$^{+}$ | 18.3$^{**}$ |
| | (7.2) | (1.2) | (2.0) | (1.4) | (5.9) |
| N | 662 | 662 | 662 | 662 | 662 |
| Mean | 881.5 | 132.5 | 196.6 | 221.8 | 603.9 |
| Median | 458 | 62.6 | 112.9 | 139.2 | 318.8 |

| | Private firm equity | Change in stock holdings | Change in h.h. debt | Log of pre-entry earnings |
|---|---|---|---|---|
| $r_{i,t}$ | 1.00$^{**}$ | $-0.23^{*}$ | 1.79 | $-0.003$ |
| | (0.37) | (0.1) | (1.37) | (0.01) |
| N | 662 | 662 | 662 | 620 |
| Mean | 44.2 | -.83 | 23.37 | 13.04 |
| Median | 21.84 | 0 | -8.93 | 13.24 |

Notes: The table corresponds to Table 5 but dropping 10% of the lowest ranked firms by assets in each return bin. All monetary values are in thousands of 2010 NOK. Outcomes are winsorized at the 5th and 95th percentiles. Working capital is defined as the difference between current assets and current liabilities. Entrepreneur balance sheet outcomes in the bottom panel are relative to lagged gross financial wealth. "Private firm equity" denotes the book value of a household's holdings of private firm equity relative to lagged gross financial wealth. "Change in stock holdings" is the change in the value of the portfolio of directly held stocks given constant stock prices between $t-1$ and $t$ relative to lagged gross financial wealth. "Change in h.h. debt" is the change in total household debt over lagged gross financial wealth. The first three outcomes in the bottom table are scaled by 100 for easier interpretation. Bootstrapped standard errors in parentheses. The rows labeled P(=Table 5) report the p-values from a bootstrapped t-test of equality between the coefficients in this table and in Table 5. + denotes significance at the 10% level, * denotes significance at the 5% level, and ** denotes significance at the 1% level.

Table A.10: Firm and Entrepreneur Outcomes – Sectoral Robustness

| | Sales | Wage bill | Empl. | Value Added | VA/worker | EBITDA |
|---|---|---|---|---|---|---|
| $r_{i,t}$ | 82.4** | 13.5+ | 0.05* | 31.4* | −4.5 | 3.3 |
| | (28.6) | (7.5) | (0.02) | (15.2) | (3.8) | (2.9) |
| N | 553 | 553 | 553 | 553 | 553 | 553 |
| Mean | 2149.3 | 687.9 | 2.4 | 1247.8 | 658.1 | 167.5 |
| Median | 1383 | 555.8 | 1 | 909.8 | 495.3 | 98.2 |
| P(=Table 5) | 0.64 | 0.75 | 0.70 | 0.75 | 0.73 | 0.79 |

| | Tot. assets | Fixed assets | Wk. Cap. | Equity | Tot. Liab. |
|---|---|---|---|---|---|
| $r_{i,t}$ | 27.7** | 3.0* | 5.0** | 3.1* | 23.3** |
| | (8.9) | (1.2) | (1.9) | (1.5) | (7.4) |
| N | 553 | 553 | 553 | 553 | 553 |
| Mean | 785.3 | 133.3 | 118.3 | 177.7 | 573.1 |
| Median | 458 | 69.5 | 50.7 | 136.3 | 318.8 |
| P(=Table 5) | 0.65 | 0.80 | 0.61 | 0.96 | 0.69 |

| | Private firm equity | Change in stock holdings | Change in h.h. debt | Log of pre-entry earnings |
|---|---|---|---|---|
| $r_{i,t}$ | 1.01* | −0.31** | 3.17* | −0.01 |
| | (0.46) | (0.11) | (1.36) | (0.01) |
| N | 553 | 553 | 553 | 521 |
| Mean | 43.03 | -2.11 | 11.17 | 13.08 |
| Median | 21.84 | 0 | -16.97 | 13.12 |
| P(=Table 5) | 0.82 | 0.85 | 0.36 | 0.54 |

Notes: The table corresponds to Table 5 except that the following sectors are removed: Electricity, gas, steam, and air conditioning supply, Water supply, Mining and quarrying, Financial and insurance activities, Agriculture, forestry and fishing, and Real estate activities, Other service activities and Administrative and support service activities. All monetary values are in thousands of 2010 NOK. Outcomes are winsorized at the 5th and 95th percentiles. Working capital is defined as the difference between current assets and current liabilities. Entrepreneur balance sheet outcomes in the bottom panel are relative to lagged gross financial wealth. "Private firm equity" denotes the book value of a household's holdings of private firm equity relative to lagged gross financial wealth. "Change in stock holdings" is the change in the value of the portfolio of directly held stocks given constant stock prices between $t-1$ and $t$ relative to lagged gross financial wealth. "Change in h.h. debt" is the change in total household debt over lagged gross financial wealth. The first three outcomes in the bottom table are scaled by 100 for easier interpretation. The results are based on the selection correction procedure described in Section 6. Bootstrapped standard errors in parentheses. The rows labeled P(=Table 5) report the p-values from a bootstrapped t-test of equality between the coefficients in this table and in Table 5. + denotes significance at the 10% level, * denotes significance at the 5% level, and ** denotes significance at the 1% level.

# B Model Appendix

In this appendix, we present the details omitted from Section 2. Appendix B.1 describes two example models both of which satisfy Assumption (M): one with financial frictions and one in which entrepreneurship provides non-pecuniary benefits. We also show that the two models differ in terms of their predictions for the effect of wealth on profits, which we estimate in our empirical analysis to differentiate the two models. Appendix B.2 shows that our analysis is robust to allowing for ex-post residuals that might affect firm size and profits. Appendix B.3 analyzes a dynamic extension to motivate our regression specification with portfolio returns, and to illustrate the asymmetric effects of positive vs negative returns on entry. Finally, Appendix B.4 presents the proofs omitted from the main text.

## B.1 Models that Satisfy Assumption (M)

Consider the entry model described in the main text. Specifically, a continuum of individuals $i$ differ in productivity $z_i$ and initial assets $a_i$ (along with observable covariates $x_i$). Conditional on entry, individuals' profits, capital, and non-pecuniary benefit from entrepreneurship are given by functions that depend only on their productivity and assets, $\pi_i = \pi(z_i, a_i), k_i = k(z_i, a_i), u_i^e = u^e(z_i, a_i)$. Individuals enter if their profits and non-pecuniary benefits from entrepreneurship exceed their reservation wage

$$\pi(z_i, a_i) + u^e(z_i, a_i) \geq w(z_i).$$

In the main text, we show that if $\pi(\cdot), k(\cdot), u^e(\cdot)$ and $w(\cdot)$ satisfy Assumption (M), which we reproduce here, then the model satisfies a rank preservation property that enables us to match entrants by productivity (without directly observing productivity).

**Assumption (M).** $\frac{d(\pi(z_i,a_i)+u^e(z_i,a_i))}{da_i} \geq 0$ and $\frac{d(\pi(z_i,a_i)+u^e(z_i,a_i)-w(z_i))}{dz_i} > 0$, $\frac{dk(z_i,a_i)}{dz_i} > 0$.

We next describe two example economies that satisfy Assumption (M) but differ in terms of their predictions for $\frac{d\pi(z_i,a_i)}{da_i}$. In both cases, we assume the reservation wage does not depend on productivity, $w(z_i) = w$. We can allow productivity to increase wages to some extent, as long as the effect of productivity on wages is smaller than its effect on the net gain from entrepreneurship (which is strictly positive in both of our examples).

### B.1.1 Model with Financial Frictions

Suppose there is no non-pecuniary benefit from entrepreneurship, $u^e(z_i, a_i) = 0$, but the entrepreneur might face financial frictions. In particular, entrepreneurs maximize their profits and they enter into business only if their maximum potential profit exceeds their reservation wage, $\pi(z_i, a_i) > w(z_i) = w$. We next describe the entrepreneur's production technology subject to financial frictions. We characterize the size and profit functions and show that they satisfy Assumption (M).

Suppose that if individual $i$ starts a business, she produces according to the Cobb-Douglas technology

$$f(k;z_i) = z_i k^{\alpha} - \xi k - \kappa(z_i).$$

Here, $\xi$ is the rental rate of capital and $\kappa(z_i)$ is a fixed entry cost. We assume entry costs are weakly decreasing in productivity $\kappa'(z_i) \leq 0$ (more productive individuals are able to reduce the fixed costs). For now, we assume capital is the only factor of production, which simplifies the algebra. The analysis can be extended to include labor.[1]

We capture financial constraints with a working capital channel. Specifically, the costs, $\xi k + \kappa(z_i)$, must be paid up front. The individual can use internal resources, $a_i$, to cover some of this cost. Therefore, the individual's borrowing need is $b = \max(0, \xi k + \kappa(z_i) - a_i)$. Individuals can obtain costly outside financing. Borrowing $b \geq 0$ costs $\phi_i(b)$ where $\phi_i(0) = 0$, $\phi_i'(b) > 0$ and $\phi_i''(b) > 0$. For simplicity, we work with the quadratic function, $\phi_i(b) = \phi(z_i)\frac{b^2}{2}$ where $\phi(z_i)$ is a constant. The quadratic functional form is not necessary for the qualitative results. Importantly, we assume the cost of financing, $\phi(z_i)$, is *weakly decreasing* in $z_i$.[2] More productive entrepreneurs obtain financing at a lower cost. This can be microfounded with a model in which there is default due to ex-post productivity shocks and outside financing costs depend on the likelihood of default (e.g., a costly-state verification model). In that type of model, a higher ex-ante productivity translates into a lower probability of default and therefore lower outside financing costs.[3]

With these assumptions, an entrepreneur that chooses to enter with capital $k$ makes profits

$$\Pi(k;z_i,a_i) = z_i k^{\alpha} - \xi k - \kappa(z_i) - \phi(z_i)\frac{\max(0, \xi k + \kappa(z_i) - a_i)^2}{2}.$$

---

[1] If the labor bill is not subject to the working capital constraint (that we describe below), then labor is straightforward to incorporate. Specifically, suppose the production function is $\tilde{z}_i k^{\tilde{\alpha}} l^{\theta} - \kappa(z_i)$ and labor is supplied at a competitive wage $w$. Then, the entrepreneur always chooses the optimum amount of labor conditional on the other factors: that is

$$l^{opt} = \arg\max_{l} \tilde{z}_i k^{\tilde{\alpha}} l^{\theta} - \kappa(z_i) - \xi k - wl.$$

The solution is given by $l^{opt} = \left(\frac{\tilde{z}_i \theta k^{\tilde{\alpha}}}{w}\right)^{1/1-\theta}$. Substituting this back into the production function, we obtain

$$\begin{aligned} f(k;z_i) &= z_i k^{\alpha} - \kappa(z_i) - \xi k \\ \text{where } z_i &= \tilde{z}_i \left(\frac{\tilde{z}_i \theta}{w}\right)^{\theta/1-\theta} \text{ and } \alpha = \frac{\tilde{\alpha}}{1-\theta}. \end{aligned}$$

If labor is also subject to a working capital constraint, then incorporating labor would leave the results qualitatively unchanged but the algebra would be more complicated, since the firm would be optimizing over two factors.

[2] For the results of Lemma 1 we need to assume that either $\kappa(z_i)$ or $\phi(z_i)$ are strictly decreasing in $z_i$.

[3] We abstract away from asymmetric information on the ex-ante productivity $z_i$.

The entrepreneur's optimal profit and size (conditional on entry) are given by

$$\begin{aligned} \pi(z_i, a_i) &= \max_k \Pi(k; z_i, a_i) \\ k(z_i, a_i) &= \arg\max_k \Pi(k; z_i, a_i). \end{aligned}$$

The following result characterizes the comparative statics of the solution. The result also implies that this model satisfies Assumption (M) (recall that we assume the reservation wage is constant, $w(z_i) = w$).

**Lemma 1.** *Greater productivity strictly increases profits and size,* $\frac{d\pi(z_i,a_i)}{dz_i} > 0, \frac{dk(z_i,a_i)}{dz_i} > 0$. *In addition, greater initial assets weakly increase profits and size* $\frac{d\pi(z_i,a_i)}{da_i} \geq 0, \frac{dk(z_i,a_i)}{da_i} \geq 0$, *with strict inequality as long as the financial constraint binds.*

**Proof of Lemma.** First, consider the problem with $\phi(z_i) = 0$: the first-best case without financial constraints. Denote the solution for this case with $\pi^*(z_i), k^*(z_i)$ (note that assets do not affect the solution in this case). Note that $\frac{d\pi^*(z_i)}{dz_i} > 0, \frac{dk^*(z_i)}{dz_i} > 0$.

Now consider the original problem. Consider the funding necessary to operate the business at the first-best level

$$\overline{a}(z_i) = \xi k^*(z_i) + \kappa(z_i).$$

There are two cases to consider. If $a_i > \overline{a}(z_i)$ , the entrepreneur is effectively unconstrained and the problem is the same as the first-best case. If $a_i < \overline{a}(z_i)$, the entrepreneur is constrained. At an optimum point, the constraint binds and her profits are given by

$$\Pi(k; z_i, a_i) = z_i k^\alpha - \xi k - \kappa(z_i) - \frac{\phi(z_i)}{2}(\xi k + \kappa(z_i) - a_i)^2. \quad \text{(B.1)}$$

For the constrained case, first consider the comparative statics of the optimal size $k(z_i, a_i)$. The first order condition implies:

$$\frac{\partial \Pi(k; z_i, a_i)}{\partial k} = 0 \Longrightarrow z_i \alpha k^{\alpha-1} = \xi + \phi(z_i)(\xi k + \kappa(z_i) - a_i). \quad \text{(B.2)}$$

Implicitly differentiating with respect to $a_i$, we obtain

$$\frac{dk}{da_i} = -\frac{\frac{\partial^2 \Pi(k;z_i,a_i)}{\partial k \partial a_i}}{\frac{\partial^2 \Pi(k;z_i,a_i)}{\partial k^2}} > 0. \quad \text{(B.3)}$$

Here, the inequality follows since $\frac{\partial^2 \Pi(k;z_i,a_i)}{\partial k \partial a_i} = \phi(z_i) > 0$ and $\Pi$ is a concave function. Likewise, we have

$$\frac{dk}{dz_i} = -\frac{\frac{\partial^2 \Pi(k;z_i,a_i)}{\partial k \partial z_i}}{\frac{\partial^2 \Pi(k;z_i,a_i)}{\partial k^2}} > 0, \quad \text{(B.4)}$$

since $\frac{\partial^2\Pi(k;z_i,a_i)}{\partial k\partial z_i} = -\phi'(z_i)(\xi k+\kappa-a_i)-\phi(z_i)\kappa'(z_i)>0$.

Next consider the comparative statics of the optimal profit, $\pi(z_i,a_i)$. Using the Envelope Theorem, we obtain

$$\begin{aligned}\frac{d\pi(z_i,a_i)}{dz_i} &= \frac{\partial\Pi(k;z_i,a_i)}{\partial z_i}|_{k=k(z_i,a_i)}\\ &= k^\alpha - \frac{\phi'(z_i)}{2}(\xi k+\kappa-a_i)^2>0\\ \frac{d\pi(z_i,a_i)}{da_i} &= \frac{\partial\Pi(k;z_i,a_i)}{\partial a_i}|_{k=k(z_i,a_i)}\\ &= \phi(z_i)(\xi k+\kappa-a_i)>0.\end{aligned}$$

Combining these comparative statics for the constrained case with the earlier characterization of the unconstrained case completes the proof. □

### B.1.2 Model in which Entrepreneurship Provides Non-pecuniary Benefits

Consider the same model without financial frictions, $\phi(z_i)=0$. Instead, suppose the non-pecuniary utility from entrepreneurship is given by a function of size and consumption

$$u^e = U^e(k,c;z_i,a_i) \text{ where } c = a_i+\Pi(k;z_i).$$

Here, $c$ is consumption and $\Pi(k;z_i)$ denotes the profit function described in $(B.1)$ (we dropped the dependence on $a_i$ since $\phi(z_i)=0$). We assume the benefit from entrepreneurship satisfies

$$\frac{dU^e}{dc}>0, \frac{dU^e}{dk}\geq 0, \frac{d^2U^e}{dcdk}\geq 0.$$

These assumptions capture the idea that individuals enjoy running a (larger) business, and more so when their regular consumption is higher. We also assume $U^e$ is jointly concave in $c$ and $k$ and strictly concave in $k$. One example function that satisfies these assumptions is $U^e(k,c)=k^\gamma c^\beta$ for arbitrary $\gamma\in[0,1)$ and $\beta\in(0,1)$.

In this case, the entrepreneur solves

$$\begin{aligned}k(z_i,a_i) &= \arg\max_{k,c} c+U^e(k,c;z_i,a_i)\\ &\quad \text{s.t. } c=a_i+\Pi(k;z_i).\end{aligned} \tag{B.5}$$

The following lemma characterizes the solution and its comparative statics. The result also implies that this model satisfies Assumption (M) (recall that we assume the reservation wage is constant, $w(z_i)=w$).

**Lemma 2.** *Consider problem $(B.5)$ with the assumptions described above. The optimal size is the* *unique solution to*

$$\frac{d\Pi}{dk}\left(1+\frac{dU^e}{dc}\right)=-\frac{dU^e}{dk}. \tag{B.6}$$

*The optimal size weakly exceeds the profit-maximizing size: that is,* $k \geq k^*$ *where* $k^* = \arg\max_k \Pi(k, z_i)$. *Greater productivity increases the firm size and the total utility from entrepreneurship,* $\frac{dk(z_i,a_i)}{dz_i} > 0$, $\frac{d(\pi(z_i,a_i)+u^e(z_i,a_i))}{dz_i} > 0$. *Greater wealth weakly increases the firm size and the total entrepreneurship utility,* $\frac{dk(z_i,a_i)}{da_i} \geq 0$, $\frac{d(\pi(z_i,a_i)+u^e(z_i,a_i))}{da_i} \geq 0$, *but it* weakly decreases *firm profits,* $\frac{d\pi(z_i,a_i)}{da_i} \leq 0$.

**Proof of Lemma.** In view of the concavity assumptions, problem $(B.5)$ has a unique solution characterized by the optimality condition $(B.6)$. Since $\frac{dU^e}{dk} \geq 0$ and $\frac{dU^e}{dc} > 0$, this condition implies $\frac{d\Pi}{dk} \leq 0$. Since $\Pi$ is strictly concave and the profit-maximizing size level $k^*$ satisfies $\frac{d\Pi(k^*)}{dk} = 0$, this also implies $k \geq k^*$.

We next establish the comparative statics. Consider $\frac{dk(z_i,a_i)}{dz_i}$. We rewrite $(B.6)$ as $\frac{d(c+U^e)}{dk} = 0$. Implicitly differentiating this expression with respect to $z_i$, we obtain

$$\frac{dk(z_i,a_i)}{dz_i}=\frac{\frac{d^2(c+U^e)}{dkdz_i}}{-\frac{d^2(c+U^e)}{dk^2}}.$$

The denominator is strictly positive. Using $c = a_i + \Pi(k; z_i)$, we calculate the numerator as

$$\frac{d^2(c+U^e)}{dkdz_i}=\frac{d^2\Pi}{dkdz_i}\left(1+\frac{dU^e}{dc}\right)+\frac{d\Pi}{dk}\frac{d^2U^e}{dc^2}\frac{d\Pi}{dz_i}+\frac{d^2U^e}{dkdc}\frac{d\Pi}{dz_i}>0.$$

Here, the inequality follows since $\frac{d^2\Pi}{dkdz_i} > 0$, $\frac{dU^e}{dc} > 0$, $\frac{d^2U^e}{dc^2} \leq 0$, $\frac{d\Pi}{dk} \leq 0$, $\frac{d\Pi}{dz_i} > 0$ and $\frac{d^2U^e}{dkdc} \geq 0$. This proves $\frac{dk(z_i,a_i)}{dz_i} > 0$.

Next consider $\frac{dk(z_i,a_i)}{da_i}$. As before, $\frac{dk(z_i,a_i)}{da_i}$ has the same sign as $\frac{d^2(c+U^e)}{dkda_i}$. We calculate

$$\frac{d^2(c+U^e)}{dkda_i}=\frac{d\Pi}{dk}\frac{d^2U^e}{dc^2}+\frac{d^2U^e}{dkdc}\geq 0.$$

Here, the inequality follows since $\frac{d\Pi}{dk} \leq 0$, $\frac{d^2U^e}{dc^2} \leq 0$ and $\frac{d^2U^e}{dkdc} \geq 0$. This implies $\frac{dk(z_i,a_i)}{da_i} \geq 0$. Since $\frac{d\Pi}{da_i} = \frac{d\Pi}{dk}\frac{dk}{da_i}$ and $\frac{d\Pi}{dk} \leq 0$, this also implies $\frac{d\pi(z_i,a_i)}{da_i} \leq 0$.

Finally, consider the comparative statics of the total utility from entrepreneurship, $\pi(z_i, a_i) + u^e(z_i, a_i)$. Increasing $z_i$ strictly increases the objective function in problem $(B.5)$ for any given choice of $k$. Therefore, it also strictly increases the maximum, which is given by $a_i + \Pi(k; z_i) + U^e(k; z_i)$. This implies $\frac{d(\pi(z_i,a_i)+u^e(z_i,a_i))}{dz_i} > 0$. The same argument also implies $\frac{d(\pi(z_i,a_i)+u^e(z_i,a_i))}{da_i} \geq 0$, completing the proof. □

## B.2 Unobserved Residual Heterogeneity

In the main text, we assumed firm size, profits, and outside options depend only on entrepreneurial productivity and assets. In practice, these variables might be heterogeneous also on other dimensions. In this appendix, we show that our selection approach is robust to allowing for residual heterogeneity under two conditions. First, we require the residual heterogeneity to be independent from initial wealth and entrepreneurial productivity conditional on observed characteristics. Second, we focus on entrants with size levels that exceed the entry cutoff by a margin. Under these assumptions, along with appropriate technical assumptions, we show that our rank matching approach still controls for average productivity and identifies the causal effect of initial wealth on firm profits and size.

Let $\overline{\delta}$ denote a vector of *ex-ante* characteristics that can influence firm size, profits, and outside options beyond entrepreneurial productivity $z$ and initial wealth $a$. For instance, $\overline{\delta}$ can correspond to industry differences in technology or wage earnings potential that is orthogonal to entrepreneurial ability. Let $\overline{\varepsilon}$ denote a vector of *ex-post* characteristics that can influence ex-post firm size and profits beyond $z$ and $a$. For instance, $\overline{\varepsilon}$ can correspond to ex-post productivity shocks or simply mistakes (relative to the optimal choice). We let $\tilde{k}\left(z,a,\overline{\delta},\overline{\varepsilon}\right), \tilde{\pi}\left(z,a,\overline{\delta},\overline{\varepsilon}\right), \tilde{u}^e\left(z,a,\overline{\delta},\overline{\varepsilon}\right)$ denote the size, profit, and entrepreneurial utility functions incorporating the ex-ante and ex-post residuals. We define the expected size, profit, and entrepreneurial utility functions as $k\left(z,a,\overline{\delta}\right) = E\left[\tilde{k}\left(z,a,\overline{\delta},\overline{\varepsilon}\right)\right], \pi\left(z,a,\overline{\delta}\right) = E\left[\tilde{\pi}\left(z,a,\overline{\delta},\overline{\varepsilon}\right)\right]$ and $\tilde{u}^e\left(z,a,\overline{\delta}\right) = E\left[\tilde{\pi}\left(z,a,\overline{\delta},\overline{\varepsilon}\right)\right]$. We let $w\left(z,\overline{\delta}\right)$ denote the outside option function that incorporates the ex-ante residuals.

Our main assumption is that $\overline{\delta}$ and $\overline{\varepsilon}$ are both independent from $(z,a)$ and from each other *conditional* on $x$. Specifically, letting $F_{\overline{\delta},\overline{\varepsilon},z,a}$ denote the joint cumulative distribution of these variables, we strengthen Assumption (CIA) as follows.

**Assumption (CIA$^{\mathrm{R}}$).** As before, $z$ and $a$ are independent conditional on $x$. In addition, residual characteristics $\overline{\delta}$ and $\overline{\varepsilon}$ are independent from $(z,a)$ and from each other conditional on $x$, that is: $F_{\overline{\delta}}\left(\overline{\delta}|\overline{\varepsilon},z,a,x\right) = F_{\overline{\delta}}\left(\overline{\delta}|x\right)$ and $F_{\overline{\varepsilon}}\left(\overline{\varepsilon}|\overline{\delta},z,a,x\right) = F_{\overline{\varepsilon}}\left(\overline{\varepsilon}|x\right)$.

For the rest of the appendix, we focus on individuals with a given $x$. At the expense of additional analytical complexity, we could combine Assumption (CIA$^{\mathrm{R}}$) and propensity score reweighting to generalize our results for the unconditional case, similar to how we extend Proposition 2 to Proposition 3 in the main text.

Focusing on individuals with a given $x$ enables us to drop $x$ from the notation. In particular, we define the *unobserved* residuals as the surprise component of the residuals given $x$:

$$\delta = \overline{\delta} - E\left[\overline{\delta}|x\right] \text{ and } \varepsilon = \overline{\varepsilon} - E\left[\overline{\varepsilon}|x\right].$$

Note that unobserved residuals have a zero mean by definition. We also redefine the functions

$\tilde{k}, \tilde{\pi}, k, \pi, w$ in terms of $\delta, \varepsilon$ as opposed to $\overline{\delta}, \overline{\varepsilon}$, for instance:

$$\tilde{k}(z, a, \delta, \varepsilon) = \tilde{k}\left(z, a, \delta + E\left[\overline{\delta}|x\right], \varepsilon + E\left[\overline{\varepsilon}|x\right]\right).$$

Note that these redefined functions implicitly depend on the covariates $x$. Likewise, we define the marginal distributions of unobserved residuals as

$$F_{\delta}(\delta) = F_{\overline{\delta}}\left(\delta + E\left[\overline{\delta}|x\right]|x\right) \text{ and } F_{\varepsilon}(\varepsilon) = F_{\overline{\varepsilon}}\left(\varepsilon + E\left[\overline{\varepsilon}|x\right]|x\right).$$

We assume the distributions $F_{\delta}(\delta), F_{\varepsilon}(\varepsilon)$ have bounded supports denoted by $\mathcal{D} = \left[-\overline{\delta}, \overline{\delta}\right]^{\mathcal{N}_{\delta}}$ and $\mathcal{E} = \left[-\overline{\varepsilon}, \overline{\varepsilon}\right]^{N_{\varepsilon}}$ (where $N_{\delta}$ and $N_{\varepsilon}$ denote the size of the vectors $\delta$ and $\varepsilon$).

With residual heterogeneity, the individual chooses to enter into business, $E = 1$, if the following condition holds:

$$\pi(z, a, \delta) + u^{e}(z, a, \delta) \geq w(z, \delta).$$

We assume these functions and $k(z, a, \delta)$ satisfy the following version of assumption (M).

**Assumption (M$^{\mathrm{R}}$).** $\frac{d(\pi(z,a,\delta)+u^{e}(z,a,\delta))}{da} \geq 0$ and $\frac{d(\pi(z,a,\delta)+u^{e}(z,a,\delta)-w(z,\delta))}{dz} > 0$, $\frac{dk(z,a,\delta)}{dz} > 0$.

With this assumption, a version of Proposition 1 still holds. Specifically, for each level of ex-ante residuals $\delta$, there is a threshold level of productivity $\overline{z}(a, \delta)$ such that an individual enters into business if $z \geq \overline{z}(a, \delta)$. Since the total benefit from entry is increasing in initial assets $a$, we also have that $\overline{z}(a, \delta)$ is weakly decreasing in $a$. Therefore, for an initial wealth level $a$, the fraction of entrants is given by

$$e(a) = \int_{\mathcal{D}} \int_{z \geq \overline{z}(a,\delta)} dF_{z}(z)\, dF_{\delta}(\delta)$$

and it is weakly increasing in $a$, generalizing Proposition 1 to this case. Intuitively, residual heterogeneity that is uncorrelated with initial wealth does not change the prediction that higher initial wealth increases entry.

Our results on the effect of wealth on business characteristics require further adjustment. We can no longer use size to control for productivity, because size depends on both productivity and residuals. We next show how to adjust our procedure to account for residual heterogeneity.

A first issue is that unlike in the main text there is not a single productivity level corresponding to a given size level. Specifically, for a given $k$ and $a$, this size might be chosen by entrepreneurs with lower $z$ and higher $\varepsilon, \delta$ or higher $z$ and lower $\varepsilon, \delta$ (assuming the convention that higher $\varepsilon, \delta$ increases $k$). To address this issue, we define the average productivity of the entrants that have initial wealth $a$ and choose size $k$ as:

$$z(k, a) = E\left[z|E = 1 \text{ and } \tilde{k}(z, a, \delta, \varepsilon) = k\right]. \tag{B.7}$$

Likewise, we also define the average profits for this group as

$$\pi(k,a) = E\left[\pi(z,a)\,|E=1 \text{ and } \tilde{k}(z,a,\delta,\varepsilon)=k\right].$$

A more major issue is that the distribution of size does not fully inform about the distribution of the (unobserved) productivity. To see this, observe that the fraction of entrants with size that exceeds $k$ are now given by:

$$e\left(a,\tilde{k}\geq k\right) = \int_{\mathcal{D}}\int_{\mathcal{E}}\int_{E=1,\tilde{k}(z,a,\delta,\varepsilon)\geq k} dF_z(z)\,dF_\varepsilon(\varepsilon)\,dF_\delta(\delta).$$

Unlike in the main text, this fraction is not necessarily equal to $\Pr(z \geq z(k,a)) = \int_{z\geq z(k,a)} dF_z(z)$, since it is also influenced by the distributions of $\delta$ and $\varepsilon$. Consequently, we cannot use the fractions $e\left(a,\tilde{k}\geq k\right)$ to match high and low initial wealth groups by productivity.

To make progress, we impose additional structure.

**Assumption (L).** For a given asset level $a$ the size and the profit functions are linear in the remaining variables:

$$\tilde{k}(z,a,\delta,\varepsilon) = K(a) + K_z(a)\,z + K_\delta(a)\,\delta + K_\varepsilon(a)\,\varepsilon, \quad \text{(B.8)}$$

$$\tilde{\pi}(z,a,\delta,\varepsilon) = \Pi(a) + \Pi_z(a)\,z + \Pi_\delta(a)\,\delta + \Pi_\varepsilon(a)\,\varepsilon. \quad \text{(B.9)}$$

Here, $K(a), K_z(a), \Pi(a), \Pi_z(a)$ denote arbitrary constants and $K_\delta(a), \Pi_\delta(a), K_\varepsilon(a), \Pi_\varepsilon(a)$ denote conforming vectors. We assume $K_z(a), \Pi_z(a) > 0$ (consistent with Assumption (M)) and $K_\delta(a), K_\varepsilon(a) \geq 0$ (Convention).

**Assumption (U).** We also assume $z$ has a uniform distribution: that is, $dF_z(\cdot)$ is constant.

These linearity and uniform-distribution assumptions enable us to obtain certainty-equivalence results and generalize our rank-matching approach, as long as we focus on size levels that exceed the entry cutoff by some margin.

Near the entry cutoffs, our approach might result in biased estimates due to selection driven by the entry decision. To see the issue, fix an initial wealth level $a$. Consider the cutoff productivity corresponding to this wealth level and the average residual, $\overline{z}(a,\mathbf{0})$. Let $\overline{k}(a) = K(a) + K_z(a)\,\overline{z}(a,\mathbf{0})$ denote the corresponding expected size. Consider the *entrants* with size $\overline{k}(a)$. The average productivity of these entrants is not necessarily equal to $\overline{z}(a,\mathbf{0})$, because some of the lower productivity individuals that would choose this size might not enter. Specifically, the lowest-productivity individual that chooses this size has high residuals $\overline{\delta}, \overline{\varepsilon}$ and low productivity given by

$$\begin{aligned}\underline{z} &= \frac{\overline{k}(a) - K(a) - K_\delta(a)\,\overline{\delta} - K_\varepsilon(a)\,\overline{\varepsilon}}{K_z(a)} \\ &= \overline{z}(a,\mathbf{0}) - \frac{K_\delta(a)\,\overline{\delta} + K_\varepsilon(a)\,\overline{\varepsilon}}{K_z(a)}.\end{aligned}$$

This individual will enter only if her productivity satisfies:

$$\overline{z}(a, \mathbf{0}) - \frac{K_\delta(a)\,\overline{\delta} + K_\varepsilon(a)\,\overline{\varepsilon}}{K_z(a)} > \overline{z}\left(a, \overline{\delta}\right). \tag{B.10}$$

This condition is violated when there is only ex-post heterogeneity ($\overline{\delta} = 0$ and $\overline{\varepsilon} > 0$). The condition is also violated when there is ex-ante heterogeneity that affects size choice but not the entry decisions, $\overline{z}\left(a, \overline{\delta}\right) = \overline{z}(a, \mathbf{0})$. In these cases, and in many others, the average productivity of entrants with size $\overline{k}(a)$ will be *greater* than $\overline{z}(a, \mathbf{0})$. Intuitively, since size is driven by both residuals and productivity, and higher productivity agents are more likely to enter, the entrants with a given size will tend to be selected relatively more on productivity. Since this selection is unobserved and might be different for high and low-wealth groups, we cannot directly apply our procedure.

To circumvent this selection problem, we consider size levels that exceed the (average) entry cutoff by a sufficient margin so that all individuals that would choose this size enter. Formally, we assume:

$$\begin{aligned} k &> \overline{k}(a) + m(a) \qquad \text{(B.11)} \\ \text{where } \overline{k}(a) &= K(a) + K_z(a)\,\overline{z}(a, \mathbf{0}) \\ \text{and } m(a) &= K_\varepsilon(a)\,\overline{\varepsilon} + K_\delta(a)\,\overline{\delta} + K_z(a)\left(\overline{z}\left(a, \overline{\delta}\right) - \overline{z}(a, \mathbf{0})\right). \end{aligned}$$

With this assumption, condition $(B.10)$ holds so the selection problem does not arise. Note also that the required margin $m(a)$ is decreasing in residual heterogeneity and limits to zero as $\overline{\varepsilon}, \overline{\delta} \to 0$. In the empirical implementation, we set $m(a)$ to a sizeable fraction of $\overline{k}(a)$. This is sufficient to address the selection problem as long as the residual heterogeneity is not too large.

Consider a size level $k > \overline{k}(a) + m(a)$. We next show that the average productivity of entrants with size $k$ satisfies a certainty-equivalence property:

$$z(k, a) = E\left[z | E = 1, \tilde{k}(z, a, \delta, \varepsilon) = k\right] = \frac{k - K(a)}{K_z(a)}. \tag{B.12}$$

In particular, $z(k, a)$ is equal to the productivity of the entrant with size $k$ for which the residuals are equal to their means, $\delta, \varepsilon = 0$ (see $(B.8)$). To see this, first note that $z(k, a) = E\left[z | \tilde{k}(z, a, \delta, \varepsilon) = k\right]$: we can drop the conditioning on $E = 1$ since $k > \overline{k}(a) + m(a)$ $(B.7)$. Next substitute for $\tilde{k}(z, a, \delta, \varepsilon)$ from Assumption (L) and calculate the conditional expectation as:

$$z(k, a) = \frac{\int_\delta \int_\varepsilon z \Pr(\delta) \Pr(\varepsilon) \Pr\left(z = \frac{k - K(a) - K_\delta(a)\delta - K_\varepsilon(a)\varepsilon}{K_z(a)}\right) d\varepsilon d\delta}{\int_\delta \int_\varepsilon \Pr(\delta) \Pr(\varepsilon) \Pr\left(z = \frac{k - K(a) - K_\delta(a)\delta - K_\varepsilon(a)\varepsilon}{K_z(a)}\right) d\varepsilon d\delta}$$

$$
\begin{aligned}
&= \frac{\int_\delta \int_\varepsilon \frac{k-K(a)-K_\delta(a)\delta-K_\varepsilon(a)\varepsilon}{K_z(a)} dF_\delta(\delta)\, dF_\varepsilon(\varepsilon)}{\int_\varepsilon dF_\delta(\delta)\, dF_\varepsilon(\varepsilon)} \\
&= \frac{k-K(a)}{K_z(a)}.
\end{aligned}
$$

Here, the second line uses the uniform distribution assumption ($\Pr(z) = dF_z$ is constant) and the last line uses the normalizations, $E[\delta] = E[\varepsilon] = 0$. This establishes the certainty-equivalence property in $(B.12)$.

We next show that the average profits for entrants with size $k$ (that exceeds $\overline{k} + m$) satisfies a similar certainty-equivalence property:

$$
\pi(k,a) = \tilde{\pi}(z(k,a), a, \mathbf{0}, \mathbf{0}) = \Pi(a) + \Pi_z(a)\left(\frac{k-K(a)}{K_z(a)}\right). \tag{B.13}
$$

In particular, $\pi(k,a)$ is equal to the profits earned by an entrant for which the productivity is $z(k,a) = \frac{k-K(a)}{K_z(a)}$ and the residuals are equal to their means, $\delta, \varepsilon = 0$ (see $(B.9)$). To show this, we follow similar steps as above to calculate

$$
\begin{aligned}
\pi(k,a) &= \frac{\int_\delta \int_\varepsilon [\Pi(a) + \Pi_z(a) z + \Pi_\varepsilon(a)\varepsilon] \Pr(\delta)\Pr(\varepsilon)\Pr\left(z = \frac{k-K(a)-K_\varepsilon(a)\varepsilon}{K_z(a)}\right) d\varepsilon d\delta}{\int_\delta \int_\varepsilon \Pr(\delta)\Pr(\varepsilon)\Pr\left(z = \frac{k-K(a)-K_\delta(a)\delta-K_\varepsilon(a)\varepsilon}{K_z(a)}\right) d\varepsilon d\delta} \\
&= \int_\delta \int_\varepsilon \left(\Pi(a) + \Pi_z(a)\left(\frac{k-K(a)-K_\delta(a)\delta-K_\varepsilon(a)\varepsilon}{K_z(a)}\right) + \Pi_\varepsilon(a)\varepsilon\right) dF_\delta(\delta)\, dF_\varepsilon(\varepsilon) \\
&= \Pi(a) + \Pi_z(a)\left(\frac{k-K(a)}{K_z(a)}\right).
\end{aligned}
$$

This establishes the certainty-equivalence property in $(B.13)$.

Finally, we show that the fraction of entrants with size that exceeds $k$ satisfies a similar certainty-equivalence property:

$$
e\left(a, \tilde{k} \geq k\right) = \int_{z \geq z(k,a)} dF_z(z). \tag{B.14}
$$

In particular, the fraction $e\left(a, \tilde{k} \geq k\right)$ is equal to the fraction that would obtain in a model in which the residuals are constant and equal to their means, $\delta, \varepsilon = 0$. To see this, note that:

$$
\begin{aligned}
e\left(a, \tilde{k} \geq k\right) &= \int_\varepsilon \int_{K(a)+K_z(a)z+K_\varepsilon(a)\varepsilon \geq k} dF_z(z)\, dF_\delta(\delta)\, dF_\varepsilon(\varepsilon) \\
&= \int_\varepsilon \Pr\left(z \geq \frac{k-K(a)-K_\delta(a)\delta-K_\varepsilon(a)\varepsilon}{K_z(a)}\right) dG(\varepsilon) \\
&= \Pr\left(z \geq \frac{k-K(a)}{K_z(a)}\right) = \int_{z \geq z(k,a)} dF_z(z).
\end{aligned}
$$

Here, the last line uses the observation that the inverse cumulative distribution function $\Pr\left(z \geq \frac{k-K(a)-K_\delta(a)\delta-K_\varepsilon(a)\varepsilon}{K_z(a)}\right)$ is a linear function of its argument (since $z$ is uniform) along with

$E[\varepsilon] = 0$. This establishes the certainty-equivalence property in $(B.14)$.

Eq. $(B.14)$ shows that with linearity and uniform-distribution assumptions we *can* still use the fractions $e\left(a, \tilde{k} \geq k\right)$ to control for the (unobserved) average productivity, $z(k, a)$. Thus, we have the following result that generalizes Proposition 2 to the case with residual heterogeneity.

**Proposition 4** (Rank preservation with residual heterogeneity)**.** *Consider the model with (ex-ante or ex-post) residual heterogeneity. Suppose Assumptions ($M^R$), (L), and (U) hold. Consider an initial wealth level $a^L$ and a corresponding size level $k^L$. Let $z^L = z\left(k^L, a^L\right)$ denote the average productivity of entrants with $k^L$ and $a^L$. Let $a^H > a^L$ denote a higher wealth level. Suppose $k^L$ is sufficiently high that the following inequalities both hold*

$$k^L > \overline{k}\left(a^L\right) + m\left(a^L\right) \text{ and } \tilde{k}\left(z^L, a^H, \mathbf{0}, \mathbf{0}\right) > \overline{k}\left(a^H\right) + m\left(a^H\right), \tag{B.15}$$

*where the functions $\overline{k}(\cdot)$ and $m(\cdot)$ are given by $(B.11)$. Let $\overline{k} \geq k^L$ denote the unique size level that solves,*

$$e\left(a^H, \tilde{k} \geq \overline{k}\right) = e\left(a^L, \tilde{k} \geq k^L\right). \tag{B.16}$$

*Then, $\overline{k} = \tilde{k}\left(z^L, a^H, \mathbf{0}, \mathbf{0}\right)$: that is, $\overline{k}$ is the firm size an entrant with productivity $z^L$ and average residuals $\delta, \varepsilon = 0$ would have if she had higher initial wealth (and the same productivity). Thus, comparing $\overline{k}$ and $k^L$ identifies the causal effect of initial wealth on firm size:*

$$\overline{k} - k^L = \tilde{k}\left(z^L, a^H, \mathbf{0}, \mathbf{0}\right) - \tilde{k}\left(z^L, a^L, \mathbf{0}, \mathbf{0}\right). \tag{B.17}$$

*Likewise, comparing the average profits of the high-wealth entrants with size $\overline{k}$ with the average profits of the low-wealth entrants with size $k^L$ identifies the causal effect of initial wealth on firm profits for an entrant with productivity $z^L$ and average residuals $\delta, \varepsilon = 0$:*

$$\pi\left(\overline{k}, a^H\right) - \pi\left(k^L, a^L\right) = \tilde{\pi}\left(z^L, a^H, \mathbf{0}, \mathbf{0}\right) - \tilde{\pi}\left(z^L, a^L, \mathbf{0}, \mathbf{0}\right). \tag{B.18}$$

Despite residual heterogeneity, our rank-matching approach controls for average productivity and identifies the causal effects for an entrant with average productivity and average residuals. The intuition follows from the certainty-equivalence properties we have established. While the residuals shuffle the firm size (for a given wealth), they do not bias our approach in a particular direction. Some larger firms are less productive, and some smaller firms are more productive, but the firm with the average size has the same average productivity as in the baseline case with certainty. Thus, matching firms by their size rank across high and low-wealth group still controls for average productivity.

While assumptions (L) and (U) enable us to obtain exact certainty-equivalence, this intuition suggests that the result is likely to hold approximately under weaker assumptions. In numerical simulations, we verify that our rank-matching approach controls for average productivity well also when $z$ shocks follow a different distribution (e.g., a normal distribution) than uniform. In this sense, we view (L) and (U) as technical assumptions.

**Proof of Proposition 4.** Let $k^H = \tilde{k}\left(z^L, a^H, \mathbf{0}, \mathbf{0}\right) = K\left(a^H\right) + K_z\left(a^H\right) z^L$ denote the size for an entrant with initial wealth $a^H$, productivity $z^L$, and average residuals. Note that the inequalities in $(B.15)$ imply that assumption $(B.11)$ holds for both pairs $a = a^L, k = k^L$ and $a = a^H, k = k^H$. Note also that Eq. $(B.12)$ implies $z\left(k^H, a^H\right) = z^L$: the average productivity of entrants with initial assets $a^H$ and size $k^H = K\left(a^H\right) + K_z\left(a^H\right) z^L$ is equal to $z^L$.

Then, applying Eq. $(B.14)$ both pairs $a = a^L, k = k^L$ and $a = a^H, k = k^H$ we obtain

$$\begin{aligned} e\left(a^L, \tilde{k} \geq k^L\right) &= \int_{z \geq z(k^L, a^L)} dF_z(z) = \int_{z \geq z^L} dF_z(z) \\ e\left(a^H, \tilde{k} \geq k^H\right) &= \int_{z \geq z(k^H, a^H)} dF_z(z) = \int_{z \geq z^L} dF_z(z). \end{aligned}$$

These expressions imply that $k^H$ is the unique solution to $(B.16)$: that is, $\overline{k} = \tilde{k}\left(z^L, a^H, \mathbf{0}, \mathbf{0}\right)$. This in turn implies $(B.17)$.

Likewise, applying Eq. $(B.13)$ for both pairs $a = a^L, k = k^L$ and $a = a^H, k = k^H = \overline{k}$ we obtain

$$\begin{aligned} \pi\left(k^L, a^L\right) &= \tilde{\pi}\left(z\left(k^L, a^L\right), a^L, \mathbf{0}, \mathbf{0}\right) = \tilde{\pi}\left(z^L, a^L, \mathbf{0}, \mathbf{0}\right) \\ \pi\left(\overline{k}, a^H\right) &= \tilde{\pi}\left(z\left(\overline{k}, a^H\right), a^H, \mathbf{0}, \mathbf{0}\right) = \tilde{\pi}\left(z^H, a^L, \mathbf{0}, \mathbf{0}\right). \end{aligned}$$

Combining these expressions implies $(B.18)$, completing the proof. □

## B.3 Dynamic Extension

In the main text, we focus on a static model of entry into entrepreneurship. In this appendix, we consider a tractable model with multiple periods and investigate the interaction of wealth dynamics with the decision to enter into business. This dynamic setup motivates our regression specification in which we look at the effect of portfolio returns (as opposed to asset levels) on entry. It also suggests that positive portfolio returns are likely to generate a larger effect on entry than negative returns—an asymmetric effect that we investigate in our empirical analysis.

For tractability, we focus on financial frictions as the channel by which wealth affects entrepreneurship, i.e., we abstract away from non-pecuniary benefits from entrepreneurship. We also make other simplifying assumptions to keep the dynamic model tractable. Our goal is to derive *qualitative insights* that we expect to apply more broadly.

We proceed in three steps. First, we consider the dynamic problem of a worker who makes (speculative) portfolio investments in risky assets. Second, we consider the dynamic problem of an entrepreneur. Finally, we consider the choice between work and entrepreneurship and derive the implications for our empirical analysis.

### B.3.1 Worker's Dynamic Problem with Risky Portfolio Choice

Time is discrete $t \in \{0, 1, ..\}$. A worker starts with assets $a_0$ and earns a constant wage $w$ in each period $t$. The worker makes a consumption-savings decision and a portfolio decision. We let $s_{t+1}$ denote the payoff-relevant state in the next period $t + 1$ and assume it is drawn from an i.i.d. distribution over a finite set $s \in S$. There is a risk-free asset denoted by $f$ and gross-return given by $r^f$. There are also risky assets denoted by $j \in J$ and gross return given by $r^j(s)$. The worker believes $s_{t+1} = s$ with probability $\pi_s \geq 0$ where $\sum_{s \in S} \pi(s) = 1$. The worker's recursive problem can then be written as follows

$$\begin{aligned} V^W(a_t) &= \max_{c_t, \{\omega_t^j\}_j} \log c_t + \beta E_t\left[V^W(a_{t+1})\right] \quad \text{(B.19)} \\ \text{s.t. } a_{t+1} &= (a_t + w - c_t)\left[r^f + \sum_{j \in J} \omega_t^j \left(r(s) - r^f\right)\right] \geq -\frac{r^f w}{r^f - 1}. \end{aligned}$$

The worker starts the period with assets $a_t$ and wage $w$. She chooses how much to consume $c_t$ and how to invest her remaining wealth $a_t + w - c_t$. She invests a fraction of her wealth $\omega_t$ in the risky asset and keeps the rest in the risk-free asset. We allow for both short-selling and leverage subject to a natural borrowing limit.

We assume there is no arbitrage, each asset is non-redundant (that is, its return cannot be replicated using other assets), and the number of assets is the same as the number of states, $|J| + 1 = |S|$. These assumptions are stronger than necessary but they simplify the analysis as they ensure that markets are complete. Specifically, there are unique Arrow-Debreu state prices $\{q(s)\}_{s \in S}$ such that

$$\begin{aligned} \sum_{s \in S} q(s) r^f &= 1 \\ \sum_{s \in S} q(s) r^j(s) &= 1 \text{ for each } j \in J. \end{aligned}$$

Using these prices, we can rewrite the worker's problem as

$$\begin{aligned} &V^W(a_t) = \max_{c_t, \{a_{t+1}(s)\}_{s \in S}} \log c_t + \beta \sum_{s \in S} \pi(s) V^W(a_{t+1}(s)) \quad \text{(B.20)} \\ &\text{s.t. } \sum_{s \in S} a_{t+1}(s) q(s) = a_t + w - c_t. \end{aligned}$$

By varying her portfolio weights, the worker effectively purchases Arrow-Debreu securities for the continuation states subject to a budget constraint.

Next, we define the worker's lifetime wealth including the value of her human capital as

$$\overline{a}_t = a_t + w + \frac{w}{r^f} + \frac{w}{(r^f)^2} + ... = a_t + \frac{r^f w}{r^f - 1}.$$

Rewriting $(B.20)$ in total wealth $\overline{a}_t = a_t + \frac{r^f w}{r^f - 1}$, we obtain

$$V^W(\overline{a}_t) = \max_{c_t, \{\overline{a}_{t+1}(s)\}_{s \in S}} \log c_t + \beta \sum_{s \in S} \pi(s) V^W(\overline{a}_{t+1}(s)) \tag{B.21}$$
$$\text{s.t.} \sum_{s \in S} \overline{a}_{t+1}(s) q(s) = \overline{a}_t - c_t.$$

The worker's problem is equivalent to the problem of someone who has higher wealth (that includes the human capital) but receives no wage income.

To solve this problem, we conjecture that the value function takes the form

$$V^W(\overline{a}_t) = \frac{\log(\overline{a}_t)}{1-\beta} + v^W, \tag{B.22}$$

where $v^W$ denotes the value of a worker with effective unit wealth. Substituting this into $(B.19)$ and using the optimality conditions, we obtain

$$\begin{aligned} c_t &= (1-\beta)\overline{a}_t \\ \text{and } \overline{a}_{t+1}(s) &= \overline{a}_t \frac{\beta \pi(s)}{q(s)} \text{ for each } s \in S. \end{aligned} \tag{B.23}$$

The worker spends a constant fraction of her effective wealth. The worker's portfolio wealth in a particular state depends on *her* perceived discounted-probability $\beta\pi(s)$ relative to the Arrow-Debreu price $q(s)$.

To understand the wealth dynamics better, let us define *the risk-neutral probability* $\pi^*(s) = q(s) r^f$, which satisfies $\sum_s \pi^*(s) = 1$ since $\sum_s q(s) = \frac{1}{r^f}$. We can then rewrite the worker's wealth dynamics as follows:

$$\left(a_{t+1} + \frac{r^f w}{r^f - 1}\right) = \left(a_t + \frac{r^f w}{r^f - 1}\right) \beta r^f \frac{\pi(s)}{\pi^*(s)}. \tag{B.24}$$

The worker's wealth in state $s$ depends on three factors: (i) her past wealth, (ii) her discount factor relative to the risk-free rate $\beta r^f$, (iii) her perceived probability for state $s$ relative to the risk-neutral probability, $\frac{\pi(s)}{\pi^*(s)}$. On average, the worker's wealth grows if she has a relatively high propensity to save ($\beta r^f > 1$) and shrinks if she has a relatively low propensity to save ($\beta r^f < 1$). In addition, the worker's wealth also grows after realizations of states which she assigns a greater probability than the risk-neutral probability ($\frac{\pi(s)}{\pi^*(s)} > 1$), and shrinks after realizations in which she assigns a lower probability ($\frac{\pi(s)}{\pi^*(s)} < 1$). The worker adjusts her portfolio weights to achieve this outcome. For instance, if state $s$ is such that the return of a risky asset $j$ exceeds the risk-free rate,

$r^j(s) - r^f > 0$, the worker can increase her wealth in this state by choosing a positive exposure to stock $j$, $\omega_t^j > 0$.

For future reference, we also solve for the worker's unit-wealth value as

$$v^W = \frac{\log(1-\beta) + \frac{\beta}{1-\beta}\sum_{s\in S}\pi(s)\left(\log\frac{\beta\pi(s)}{q(s)}\right)}{1-\beta}. \tag{B.25}$$

Note that the unit-wealth value is increasing in the difference between the worker's discounted probability $\beta\pi(s)$ and the Arrow-Debreu price $q(s) = \pi^*(s)/r^f$.

### B.3.2 Entrepreneur's Dynamic Problem

We model the entrepreneur's dynamic problem similar to Moll (2014). An entrepreneur with wealth $a_t$ has access to a production technology

$$f(k_t, l_t; z) = (zk_t)^\alpha l_t^{1-\alpha}.$$

She can rent capital at a constant competitive rate $\xi$ subject to a collateral constraint $k_t \le \lambda a_t$ where $\lambda > 1$. She can rent labor at a constant competitive wage $w$. Combining the assumption, the entrepreneur's static problem is given by

$$\pi(z, a_t) = \max_{k_t, l_t} (zk_t)^\alpha l_t^{1-\alpha} - wl_t - \xi k_t \text{ s.t. } k_t \le \lambda a_t.$$

Substituting for the optimal labor given capital $l_t = \left(\frac{1-\alpha}{w}\right)^{1/\alpha} zk_t$, the problem becomes

$$\pi(z, a_t) = \max_{k_t \ge 0} Rzk_t - \xi k_t \text{ s.t. } k_t \le \lambda a_t.$$

where $R = \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)/\alpha}$. Note that this is a version of the static problem we have analyzed with $\alpha = 1$ (constant returns to scale), $\kappa(z_i) = 0$ (no entry costs), and outside financing costs that are zero up to size $k_t = \lambda a_t$ and becomes infinite for greater sizes $k_t > \lambda a_t$.

The entrepreneur's capital choice problem has a corner solution. If the entrepreneur is not sufficiently productive, $Rz \le \xi$, she does not produce, $k_t = 0$. Otherwise, she produces at the maximum scale $k_t = \lambda a_t$ and obtains a net profit

$$\pi(z, a_t) = \Pi_a(z)\, a_t \text{ where } \Pi_a(z) = (Rz - \xi)\lambda. \tag{B.26}$$

We assume $\Pi_a(z) > 0$ as this is the relevant case for the choice between work and entrepreneurship (in the other case, the individual would always stay a worker).

The entrepreneur starts the period with assets $a_t$. After operating her business, she receives period profits $\pi(z, a_t)$. At the end of the period, she makes a consumption-savings decision out of wealth $a_t + \pi(z, a_t)$ similar to the worker. For symmetry, we assume the entrepreneur makes a portfolio choice among the same assets as before. Her dynamic problem can then be written as the following analogue of $(B.21)$

$$V^E(a_t) = \max_{c_t, \{a_{t+1}(s)\}_{s \in S}} \log c_t + \beta \sum_{s \in S} \pi(s) V^E(a_{t+1}(s)) \tag{B.27}$$
$$\text{s.t.} \sum_{s \in S} a_{t+1}(s) q(s) = (1 + \Pi_a(z)) a_t - c_t.$$

To solve this problem, we conjecture that the value function takes the form (cf. $(B.22)$)

$$V^E(a_t) = \frac{\log(a_t)}{1-\beta} + v^E, \tag{B.28}$$

where $v^E$ denotes the value of an entrepreneur with unit wealth. Substituting this into $(B.27)$ and using the optimality conditions, we obtain

$$\begin{aligned} c_t &= (1-\beta)(1 + \Pi_a(z)) a_t \\ \text{and } a_{t+1}(s) &= (1 + \Pi_a(z)) a_t \frac{\beta \pi(s)}{q(s)} \text{ for each } s \in S. \end{aligned}$$

Similar to the worker, the entrepreneur takes risky positions in financial markets that reflects her beliefs. In equilibrium, asset growth is driven by the net profits, as well as the financial market forces that we discussed earlier.

Using the solution, we also solve for the entrepreneur's unit-wealth value as

$$\begin{aligned} v^E &= \frac{\log(1-\beta) + \frac{\log(1+\Pi_a(z))}{1-\beta} + \frac{\beta}{1-\beta} \sum_{s \in S} \pi(s) \left( \log \frac{\beta \pi(s)}{q(s)} \right)}{1-\beta} \\ &= v^W + \frac{\log(1 + \Pi_a(z))}{(1-\beta)^2}. \end{aligned} \tag{B.29}$$

Note that $v^E > v^W$: for the same effective wealth, the entrepreneur receives greater value because she uses her wealth to increase the size of her business, profits, and consumption.

### B.3.3 Choice Between Work and Entrepreneurship

Consider an individual that can choose between work and entrepreneurship. We start with the case in which the individual has a choice *only at the beginning of period* 0*:* once she makes the choice, she remains a worker or an entrepreneur in all future periods. Given an initial wealth $a_0$, the individual chooses entrepreneurship as long as the value from entrepreneurship exceeds the value from work

$$E_0 = 1 \text{ iff } V^E(a_0) \geq V^W(a_0).$$

Using Eqs. $(B.22 - B.25)$ and $(B.28 - B.29)$, and canceling $v^W$ from both sides, we write this condition as

$$\frac{\log(a_0)}{1-\beta} + \frac{\log(1+\Pi_a(z))}{(1-\beta)^2} \geq \frac{\log\left(a_0 + \frac{r^f w}{r^f-1}\right)}{1-\beta}$$

which implies

$$a_0 (1+\Pi_a(z))^{1/(1-\beta)} \geq a_0 + \frac{r^f w}{r^f-1}.$$

After rearranging terms, we obtain

$$a_0 \geq \overline{a}(z) \equiv \frac{\frac{r^f w}{r^f-1}}{(1+\Pi_a(z))^{1/(1-\beta)} - 1} \simeq \frac{\frac{r^f w}{r^f-1}}{\frac{\Pi_a(z)}{1-\beta}}. \tag{B.30}$$

Here, the last line linearizes the denominator and provides an approximation that holds when the net profits $\Pi_a(z)$ are relatively small. The individual is more likely to choose entrepreneurship when she has more wealth. Intuitively, as in the static model, greater wealth enables the entrepreneur to increase the scale of her business and increase her profits. Observe also that the individual is more likely to choose entrepreneurship (as reflected by a lower cutoff $\overline{a}(z)$) when the business is more profitable (higher $\Pi_a(z)$) and when the value of wage work is lower (lower $\frac{r^f w}{r^f-1}$).

Next, suppose entry is a dynamic choice: An individual that has chosen not to become an entrepreneur in the past periods may choose to do so in period $t$. We assume becoming an entrepreneur is irreversible as before. However, staying a worker has an *option value* to become an entrepreneur in the future. This option value might affect the worker's consumption-savings and portfolio choice decisions. While this option value is interesting, it complicates the analysis and it is beyond the scope of the current paper. Therefore, we make a simplifying behavioral assumption that an individual that chooses to remain a worker *assumes* she will remain a worker for the rest of her life, that is, she ignores the option value. In this case, the individual's decision is characterized by

$$E_t = 1 \text{ iff } V^E(a_t) \geq V^W(a_t) \Leftrightarrow a_t \geq \overline{a}(z), \tag{B.31}$$

where $\overline{a}(z)$ is still given by $(B.30)$.

When the individual stays a worker, she makes the same portfolio decisions as before, and her wealth evolves according to $(B.24)$. These two equations characterize the individual's dynamic choice to become an entrepreneur. As we discussed before, the individual's wealth grows more on average when she has a high propensity to save relative to the risk-free rate (higher $\beta r^f$). In addition to this average effect, her wealth grows after state realizations which she assigns a greater probability than the risk-neutral probability. If her wealth grows sufficiently to exceed $\overline{a}(z)$, the individual becomes an entrepreneur. Otherwise, she remains a worker.

**Regression specification and asymmetric returns.** We next use this characterization to motivate our empirical analysis. Using ($B$.19), the worker's wealth evolves according to

$$a_{t+1} = (a_t + w - c_t)\, r_{t+1}(s) \text{ where } r_{t+1}(s) = r^f + \sum_{j \in J} \omega_t^j \left( r(s) - r^f \right).$$

Here, $r_{t+1}(s)$ denotes the worker's realized portfolio return that depends on her (endogenous) weight on the risky portfolio. Using ($B$.23) to substitute for consumption $c_t = (1-\beta)\left(a_t + \frac{r^f w}{r^f - 1}\right)$, we obtain

$$a_{t+1} = \left(\beta a_t + w \left(\frac{\beta - 1/r^f}{1 - 1/r^f}\right)\right) r_{t+1}(s). \tag{B.32}$$

Combining this with (B.31), the worker enters into business in period $t$+1 iff the following conditions hold

$$\begin{aligned} a_t &< \overline{a}(z) \text{ and} \\ a_{t+1} &\geq \overline{a}(z) \iff r_{t+1}(s) \geq \overline{r}(z, a_t) \equiv \frac{\overline{a}(z)}{\beta a_t + w\left(\frac{\beta - 1/r^f}{1 - 1/r^f}\right)}. \end{aligned} \tag{B.33}$$

The second condition in (B.33) says that the worker enters into business if her realized portfolio return exceeds a cutoff level. The cutoff return $\overline{r}(z, a_t)$ is decreasing in both productivity and wealth $a_t$. This condition highlights that the entry decision is driven by *changes* in wealth and motivates our regression specification with the portfolio return.

The first condition in (B.33) says the entrants in period $t$ have a relatively low wealth and experience an *increase* in wealth. This condition suggests positive returns are likely to generate stronger effects than negative returns—an asymmetric effect that we explore in our empirical analysis.

To formalize the asymmetric effect, we combine $a_t < a(z)$ with ($B$.33) to provide a lower bound for the cutoff return for entry

$$\overline{r}(z, a_t) \geq \overline{r}(z) = \frac{1}{\beta + \frac{w}{\overline{a}(z)}\left(\frac{\beta - 1/r^f}{1 - 1/r^f}\right)}. \tag{B.34}$$

The lower bound for the cutoff is decreasing in the discount factor $\beta$. Consider the cases $\beta \leq 1/r^f$ and $\beta > r^f$.

When $\beta \leq 1/r^f$, the lower bound satisfies $\overline{r}(z) \geq r^f$ and the entry is characterized by:

$$r_{t+1}(s) \geq \overline{r}(z, a_t) \geq r^f.$$

That is, individual enters only if her excess return $r_{t+1}(s) - r^f$ is sufficiently positive, and she remains a worker if her excess return is either zero or negative. Hence, in this case the return has a strongly asymmetric effect, with positive excess returns generating some entry, but negative excess returns having no effect on entry (relative to a baseline with zero excess return). For intuition, observe that the individual's propensity to save is relatively low and her wealth satisfies the inequality $a_{t+1} \leq a_t \frac{r_{t+1}(s)}{r^f}$ (see Eq. $(B.32)$). In particular, her wealth increases, and she chooses to become an entrepreneur, only if she receives a sufficiently high positive excess return.

When $\beta > 1/r^f$, the lower bound is lower than the risk-free rate $\overline{r}(z) < r^f$. In this case, there are situations (with $a_t$ lower than but close to $\overline{a}(z)$) in which the individual would enter into business if her excess return was zero, but she does not enter because her excess return is negative. Intuitively, the individual's propensity to save is relatively large so that if she invested with the risk-free rate her wealth would increase and she would become an entrepreneur. Starting from this baseline, realizing a loss on her risky portfolio might prevent the individual from becoming an entrepreneur. However, even in this case, the lower bound $\overline{r}(z)$ implies highly negative excess returns generate no effect compared to a baseline negative excess return given by $\overline{r}(z) - r^f$.

## B.4 Omitted Proofs in Section 2

**Proof of Proposition 1.** Under Assumption (M), the net gain from entry, $\pi(z_i, a_i) + u^e(z_i, a_i) - w(z_i)$, is weakly increasing in $a_i$ and strictly increasing in $z_i$. The latter relation implies that for any $a_i$ there exists a threshold level $\overline{z}(a_i)$ such that an agent enters if and only if $z_i \geq \overline{z}(a_i)$. The former relation implies that $\overline{z}(a_i)$ is weakly decreasing in $a_i$, completing the proof. □

**Proof of Proposition 2.** We first claim that $\overline{k} = k\left(z, a^H\right)$ is the unique solution to Eq. (1). To this end, note that

$$\begin{aligned}
e\left(a^L, k \geq k^L | x_i\right) &= \int_{z_i \geq \overline{z}(a^L), k(z_i, a^L) \geq k^L} dF_z\left(z_i | x_i\right) \\
&= \int_{z_i \geq \overline{z}(a^L), k(z_i, a^L) \geq k(z, a^L)} dF_z\left(z_i | x_i\right) \\
&= \int_{z_i \geq z} dF_z\left(z_i | x_i\right).
\end{aligned}$$

Here, the second line substitutes $k_L = k\left(z, a^L\right)$ and the last line follows since $k\left(z_i, a^L\right)$ is monotonic in $z_i$ (and $z_i \geq z$ implies $z_i \geq \overline{z}\left(a^L\right)$). The same steps imply that for $\overline{k} = k^H\left(z, a^H\right)$ we have,

$$\begin{aligned}
e\left(a^H, k \geq \overline{k} | x_i\right) &= \int_{z_i \geq \overline{z}(a^H), k(z_i, a^H) \geq \overline{k}} dF_z\left(z_i | x_i\right) \\
&= \int_{z_i \geq \overline{z}(a^H), k(z_i, a^H) \geq k^H(z, a^H)} dF_z\left(z_i | x_i\right) \\
&= \int_{z_i \geq z} dF_z\left(z_i | x_i\right).
\end{aligned}$$

Comparing these expressions proves that $\overline{k} = k^H\left(z, a^H\right)$ solves Eq. (1). Note also that $e\left(a^H, k \geq \overline{k} | x_i\right)$ is strictly decreasing in $\overline{k}$, because the function $k\left(z_i, a^H\right)$ is strictly increasing in $z_i$ and the distribution $dF_z(z_i|x_i)$ is continuous in $z_i$. This implies that $\overline{k} = k(z, a^H)$ is the unique solution to Eq. (1).

Next consider the firm-level outcome $\overline{y}$ corresponding to the firm with size $\overline{k}$ and initial assets $a^H$. Since $\overline{k} = k(z, a^H)$, we also have $\overline{y} = y(z, a^H)$. This implies $\overline{y} - y^L = y(z, a^H) - y(z, a^L)$ and completes the proof. □

**Proof of Proposition 3.** We first claim that $\overline{k} = k(z, a^H)$ is the unique solution to (3). To this end, observe that

$$\begin{aligned}
e^*\left(a^L, k \geq k^L\right) &= \int_{x_i}\int_{z_i \geq \overline{z}(a^L), k(z_i, a^L) \geq k^L} dF_z(z_i|x_i)\,\omega(x_i)\,dF_x\left(x_i|a^L\right) \\
&= \int_{x_i}\int_{z_i \geq \overline{z}(a^L), k(z_i, a^L) \geq k^L} dF_z(z_i|x_i)\,dF_x\left(x_i|a^H\right) \\
&= \int_{x_i}\int_{z_i \geq \overline{z}(a^L), k(z_i, a^L) \geq k^L} f_z(z_i|x_i)\,f_x\left(x_i|a^H\right) dz_i dx_i \\
&= \int_{z_i \geq \overline{z}(a^L), k(z_i, a^L) \geq k^L} \left(\int_{x_i} f_z(z_i|x_i)\,f_x\left(x_i|a^H\right) dx_i\right) dz_i \\
&= \int_{z_i \geq \overline{z}(a^L), k(z_i, a^L) \geq k^L} f_z\left(z_i|a^H\right) dz_i \\
&= \int_{z_i \geq \overline{z}(a^L), k(z_i, a^L) \geq k(z, a^L)} f_z\left(z_i|a^H\right) dz_i \\
&= \int_{z_i \geq z} f_z\left(z_i|a^H\right) dz_i
\end{aligned}$$

Here, the second line uses the definition of the propensity score $\omega(x_i) = \frac{dF_x(x_i|a^H)}{dF_x(x_i|a^L)}$, the third line substitutes the PDFs corresponding to the CDFs, the fourth line changes the order of integration, the fifth line substitutes the definition of the marginal PDF $f_z\left(z_i|a^H\right) = \int_{x_i} f_z(z_i|x_i)\,f_x\left(x_i|a^H\right) dx_i$, the sixth line substitutes $k^L = k(z_i, a^L)$, and the last line follows since $k(z_i, a^L)$ is monotonic in $z_i$ (and $z_i \geq z$ implies $z_i \geq \overline{z}(a^L)$). Following similar steps, for $\overline{k} = k(z, a^H)$, we have

$$\begin{aligned}
e\left(a^H, k \geq \overline{k}\right) &= \int_{x_i}\int_{z_i \geq \overline{z}(a^H), k(z_i, a^H) \geq \overline{k}} dF_z(z_i|x_i)\,dF_x\left(x_i|a^H\right) \\
&= \int_{x_i}\int_{z_i \geq \overline{z}(a^H), k(z_i, a^H) \geq \overline{k}} f_z(z_i|x_i)\,f_x\left(x_i|a^H\right) dz_i dx_i \\
&= \int_{z_i \geq \overline{z}(a^H), k(z_i, a^H) \geq \overline{k}} \int_{x_i} f_z(z_i|x_i)\,f_x\left(x_i|a^H\right) dx_i dz_i \\
&= \int_{z_i \geq \overline{z}(a^H), k(z_i, a^H) \geq \overline{k}} f_z\left(z_i|a^H\right) dz_i \\
&= \int_{z_i \geq \overline{z}(a^H), k(z_i, a^H) \geq k(z, a^H)} f_z\left(z_i|a^H\right) dz_i \\
&= \int_{z_i \geq z} f_z\left(z_i|a^H\right) dz_i.
\end{aligned}$$

Comparing these expressions proves that $\overline{k} = k\left(z, a^H\right)$ solves (3). Note also that $e\left(a^H, k \geq \overline{k}\right)$ is strictly decreasing in $\overline{k}$, which implies that $\overline{k} = k^H\left(z, a^H\right)$ is the unique solution to Eq. (3) .

Next consider the firm-level outcome $\overline{y}$ corresponding to the firm with size $\overline{k}$ and initial assets $a^H$. Since $\overline{k} = k\left(z, a^H\right)$, we also have $\overline{y} = y\left(z, a^H\right)$. This implies $\overline{y} - y^L = y\left(z, a^H\right) - y\left(z, a^L\right)$ and completes the proof. □