

Working Paper

Macroeconomic effects of fiscal policy under an energy supply shock

Norges Bank Research

Authors:

Johan Gustafsson

Xiaofei Ma

Junior Maih

Mathias Vesterberg

Keywords

Energy price, investment subsidy,
energy voucher

Working papers fra Norges Bank, fra 1992/1 til 2009/2 kan bestilles på e-post: servicesenter@norges-bank.no

Fra 1999 og senere er publikasjonene tilgjengelige på www.norges-bank.no

Working papers inneholder forskningsarbeider og utredninger som vanligvis ikke har fått sin endelige form. Hensikten er blant annet at forfatteren kan motta kommentarer fra kolleger og andre interesserte. Synspunkter og konklusjoner i arbeidene står for forfatternes regning.

Working papers from Norges Bank, from 1992/1 to 2009/2 can be ordered by e-mail: servicesenter@norges-bank.no

Working papers from 1999 onwards are available on www.norges-bank.no

Norges Bank's working papers present research projects and reports (not usually in their final form) and are intended inter alia to enable the author to benefit from the comments of colleagues and other interested parties. Views and conclusions expressed in working papers are the responsibility of the authors alone.

ISSN 1502-8143 (online)

ISBN 978-82-8379-311-6 (online)

Macroeconomic effects of fiscal policy under an energy supply shock*

Johan Gustafsson[†] Xiaofei Ma[‡] Junior Maih[§] Mattias Vesterberg[¶]

February 10, 2024

Abstract

We study the potential effects on the real economy and welfare of four fiscal policy responses to an energy supply shock: energy vouchers to all households, only to low-income households, or to non-energy goods producers, and subsidies for investments in the energy sector. The analysis relies on a DSGE model that explicitly models the energy sector. Calibrating the model to Swedish data, our results show that the subsidy for the investment in energy sector is the most effective instrument to reduce the energy price in the short- to medium term. This policy is, however, welfare dominated by energy vouchers given to households as it immediately compensates low-income, non-saving households in the event of the shock. Giving the energy voucher to the non-energy firms prevents energy prices from falling as fast as they would without policy intervention. It is also the least desirable from a welfare perspective.

Keywords: energy price, investment subsidy, energy voucher

JEL Codes: D58, E62, H53

*This paper should not be reported as representing the views of Norges Bank. The views expressed are those of the authors and do not necessarily reflect those of Norges Bank. We are grateful for comments and suggestions provided by an anonymous referee at Norges Bank. We are also grateful for helpful discussions with Michel Juillard, Christoph Weber and Pablo Winant. The paper has also benefited from comments from the the Global Cleaner Production Conference organized by Fudan University in Shanghai, the 29th Ulvön Conference on Environmental Economics, the 29th International Conference Computing in Economics and Finance, the 5th EMCEI conference at University of Calabria, and internal seminars of ESSCA School of Management and USBE. We also benefited from funding from l'Institut Europlace de Finance du Groupe Louis Bachelier, Orientation of Financial Flow for the Energetic Transition and Ecology (ORFEE).

[†]Umeå University, E-mail: johan.a.gustafsson@umu.se

[‡]ESSCA School of Management, E-mail: xiaofei.ma@essca.fr

[§]Norges Bank, junior.maih@norges-bank.no

[¶]Umeå University, E-mail: mattias.vesterberg@umu.se

1 Introduction

During the recent years, energy prices increased substantially in Sweden and Europe. For example, the daily average price of a kWh in southern Sweden increased from less than 0.1 Euro to more than 0.5 Euro between 2021 and 2022, and the increases in price have been even larger in other European countries. This development has led to energy poverty among households and loss of profits for firms, and subsequently, policy makers have suggested a range of measures to mitigate the effects of these electricity price shocks, including subsidies, energy vouchers and price ceilings (e.g., [Ari et al. \(2022\)](#); [EU-commission \(2022b\)](#)).

In this paper, we use a dynamic stochastic general equilibrium (DSGE) model to quantify the macroeconomic effects of the suggested policies. We focus on four distinct policies that are either proposed or already implemented in Europe and/or Sweden: an energy voucher to households; an energy voucher to non-energy goods producers, and subsidies to investments in the energy sector; see, e.g., [EU-commission \(2022b\)](#). Especially the investment subsidy is a novel aspect of our policy analysis. In particular, while such policies have been highlighted by both EU and member states as a mean to reduce the exposure to high energy prices, previous literature has tended to focus on the macroeconomic effects of demand-side policies such as energy vouchers and subsidies to consumers. Since our model includes energy producers, we are able to also study supply-side policies. Here, we would like to point out that the current paper is not looking for a first-best policy; the objective of this paper is rather to quantify the macroeconomic effects of policies that are on the table, regardless of whether they are to be considered first-best policies.

The general equilibrium framework is important for several reasons. We mainly acknowledge that energy is a fundamental input in the production of goods and services, and is also a necessity for a modern lifestyle. Price fluctuations in energy thereby explicitly and implicitly affects both the demand and supply side of commodity and factor markets. Ultimately, the DSGE framework can be designed to capture both the demand-side and the supply-side

effects through household and firm resource allocation.

In more detail, we develop a model which consists of energy producers, producers of “non-energy” goods such as commodities and services, households, and a government. The energy produced is used both as input in the production of non-energy goods, and for direct consumption by the households. To keep our results general to any sort of energy supply shock, not necessarily induced by geopolitical conflicts or sudden mark-up changes, we model the energy shock as a total factor productivity (TFP) shock to the production function of the energy producer.

In the current political debate, energy poverty, equity and redistribution of wealth are highlighted as key focuses for the suggested policies. However, in Sweden, energy poverty in Sweden is mostly related to some households living hand-to-mouth. This distinguishes energy poverty in Sweden from many traditional measures of energy poverty, such as the inability to heat homes. In this sense, energy poverty in Sweden is exposed at price peaks, and the ability to dodge price peaks becomes an important quality among households to reduce their vulnerability to energy poverty (see, for example, the discussion in [von Platten \(2022b\)](#); [Antunes et al. \(2023\)](#) and https://energy-poverty.ec.europa.eu/index_en). To reflect this stylized fact, our model includes three distinct type of households: rational savers with either high or low skill, and low-skilled hand-to-mouth households. This allows our model to capture key aspects of energy poverty that has been highlighted by recent literature.

To quantify the effects of the different policies on the real economy and economic welfare, we calibrate the model to Swedish data. Sweden is an interesting case to study, given that Sweden has a very high — among the ten highest — energy intensity per capita. This is explained both by the cold and long winters, and an energy-intensive industry, and it implies that Sweden may be particularly vulnerable to energy supply shocks. Furthermore, Sweden has a large domestic energy production, is less reliant on energy imports than many other countries, and is a net exporter of electricity. This means that the Swedish government may

be able to target energy production through, e.g, subsidies, to a larger extent than many other countries.

In brief, our results reveal that the subsidy for the investment in the energy sector is the most effective instrument to reduce the energy price in the short- to medium term. This policy is, however, welfare dominated by energy vouchers given to households. The energy voucher given directly to non-energy firms has the risk to increase the energy price, and decrease considerably welfare.

Our results have important policy implications: if the government aims to reduce the energy price, then they should focus on the subsidies for energy investments. If the government considers households' welfare as the most important indicator, then they should consider energy vouchers to all households (not only to low-income households). This is apparently what the Swedish government is practicing at the moment.

The rest of the paper is structured as follows: We describe the policy background in Section 2, and review the literature on the macroeconomic effects of energy crisis and policies in Section 3. Section 4 details the model and equilibrium conditions. Section 5 outlines the calibration of the model to Swedish data, and the policy simulations are contained in Section 6. Section 7 concludes the paper.

2 Institutional context

According to the EU Commission and the UN, the very high energy prices during the recent years are primarily driven by two factors: first, there has been an increased global demand for gas following the economic recovery after Covid-19 (EU-commission (2022b); UN (2022)). This increase in demand has not been matched by an increase in supply, with consequences both in the EU and in other regions of the world. Second, the exports of gas from Russia has decreased following Russia's attack on Ukraine, with the deliberate attempt by Russia to use energy as a political weapon. Furthermore, postponed infrastructure investments

and maintenance during the pandemic has also constrained gas supply. Since gas prices are important determinants of electricity prices in most of the EU, increased gas prices has subsequently affected electricity prices. In addition, electricity prices has also increased because of, for example, low reservoir levels in the hydropower production (due to lower-than-usual precipitation) and less wind than usual. This has resulted in lower production of renewable energy in Europe. Finally, nuclear power production has decreased by approximately 20 percent in the EU.

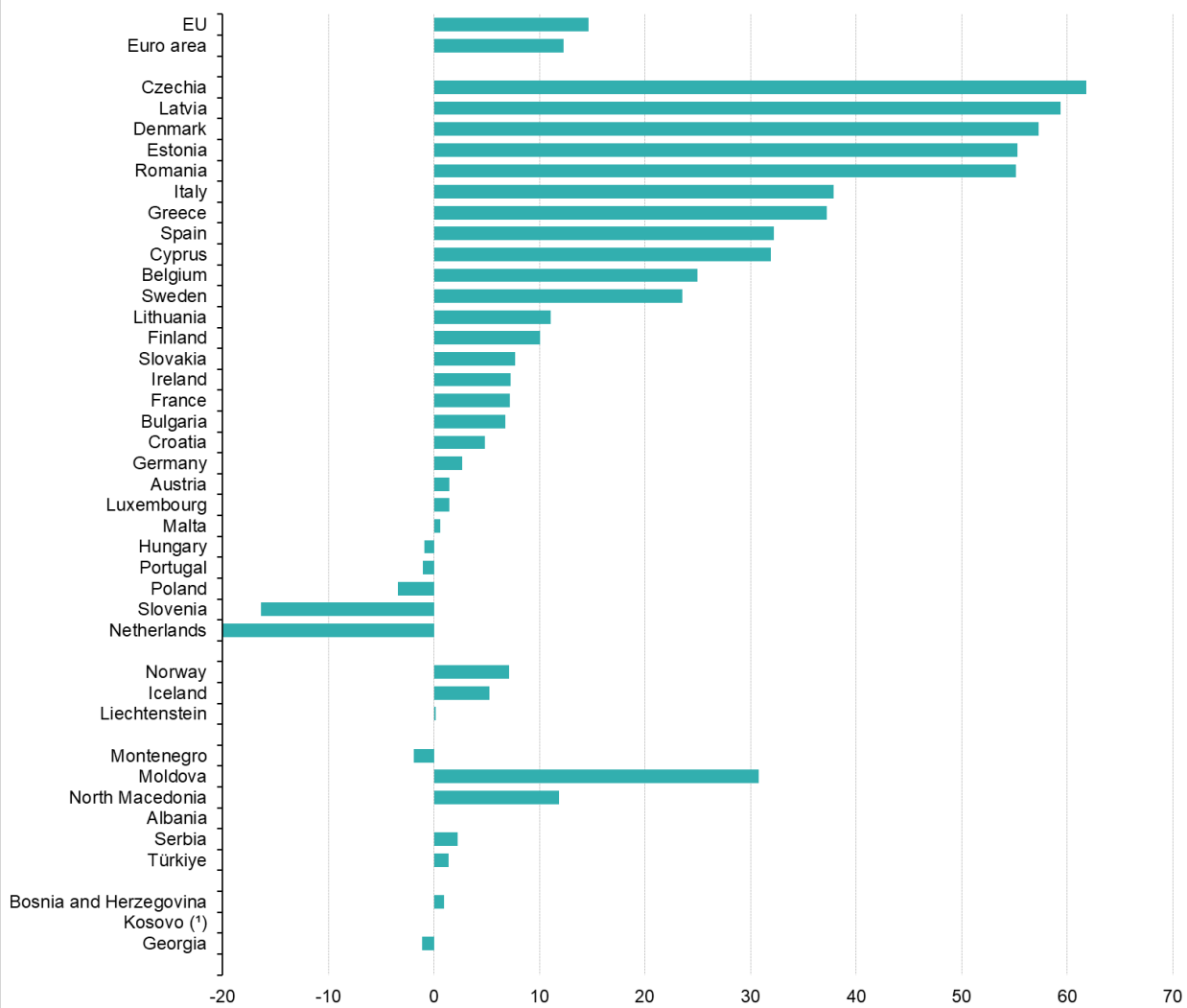
As a consequence of these events, energy prices have increased dramatically in Europe. To give a sense of magnitude, electricity prices for household consumers increased from 2021 to 2022 in all except five EU Member States, with the biggest increase in Czechia (61.8 percent), followed by Latvia (59.4 percent) and Denmark (57.3 percent); see Figure 1. Non-household consumers faced similar prices increases, as illustrated in Figure 2.¹

Similar to the rest of EU, Sweden also faced large energy price increases during 2021 and 2022, especially in the electricity market. While Sweden does not import Russian gas to produce electricity or heating to any large extent, Sweden's electricity market is integrated with northern Europe, and is therefore affected by what happens in continental Europe, and this caused Swedish electricity prices to increase substantially. The electricity spot price in southern Sweden is illustrated in Figure 3, and evidently, Sweden has had low and stable prices for a long time, but have in especially 2021 and 2022 seen both higher and more variable prices.

The recent energy price shocks have affected everyone, but low-income households may have been particularly vulnerable to such price shocks since these households typically spend a larger share of their income on energy and electricity, compared to households with higher income (see, for example, Ari et al. (2022)). To give a sense of magnitude, about nine percent of the EU population, or approximately 40 million people, were unable to keep their

¹During 2023, prices stabilized somewhat. See https://ec.europa.eu/eurostat/statistics-explained/index.php?title=Electricity_price_statistics.

Change in electricity prices for household consumers compared with previous year, same semester, first half 2022



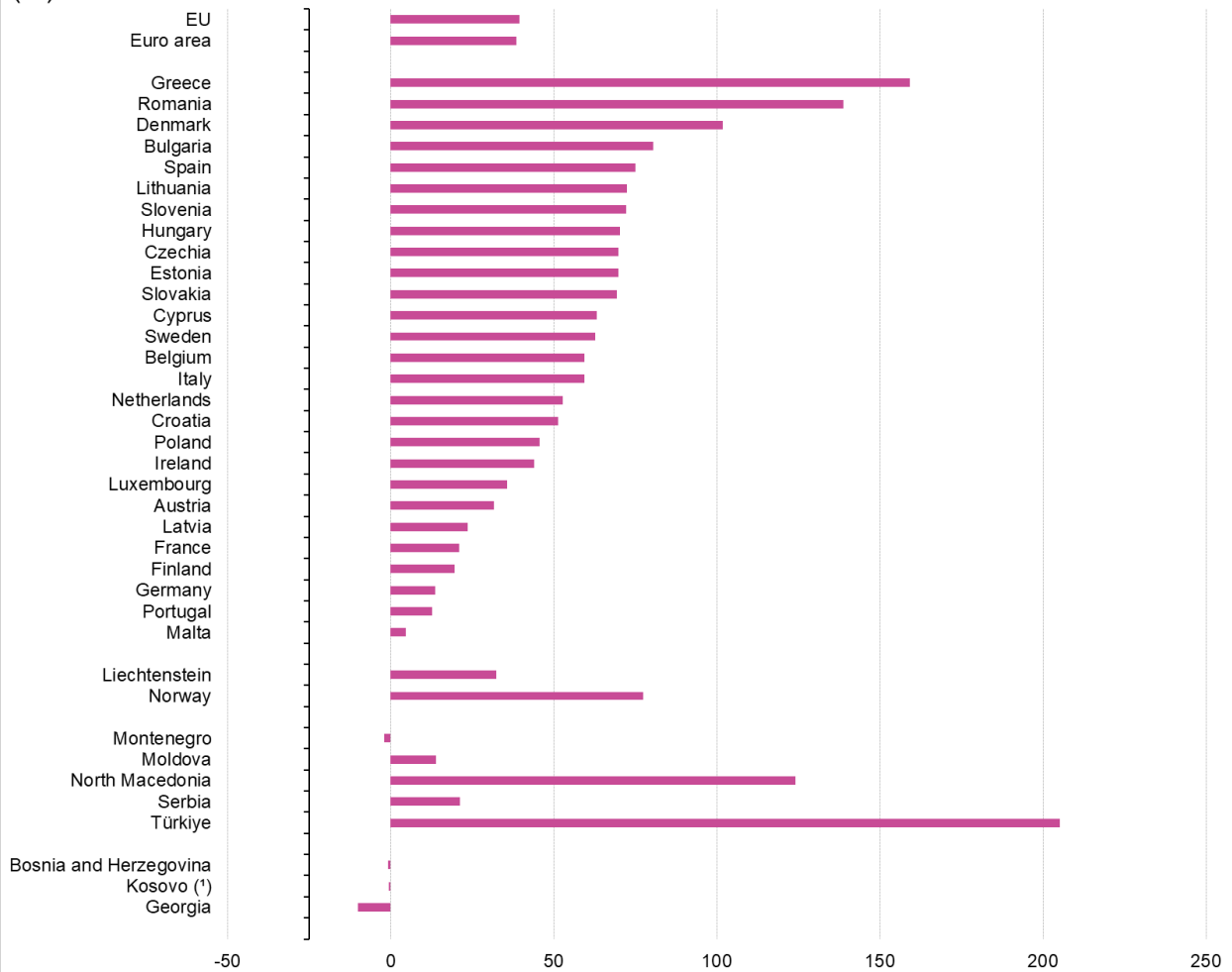
(*) This designation is without prejudice to positions on status, and is in line with UNSCR 1244/1999 and the ICJ Opinion on the Kosovo Declaration of Independence.

Source: Eurostat (online data codes: nrg_pc_204)



Figure 1: Percentage change in electricity prices for households between first half of 2021 and first half of 2022

Change in electricity prices for non-household consumers compared with previous year, same semester, first half 2022 (%)



(*) This designation is without prejudice to positions on status, and is in line with UNSCR 1244/1999 and the ICJ Opinion on the Kosovo Declaration of Independence.
 Source: Eurostat (online data codes: nrg_pc_205)



Figure 2: Percentage change in electricity prices for non-household consumers between first half of 2021 and first half of 2022

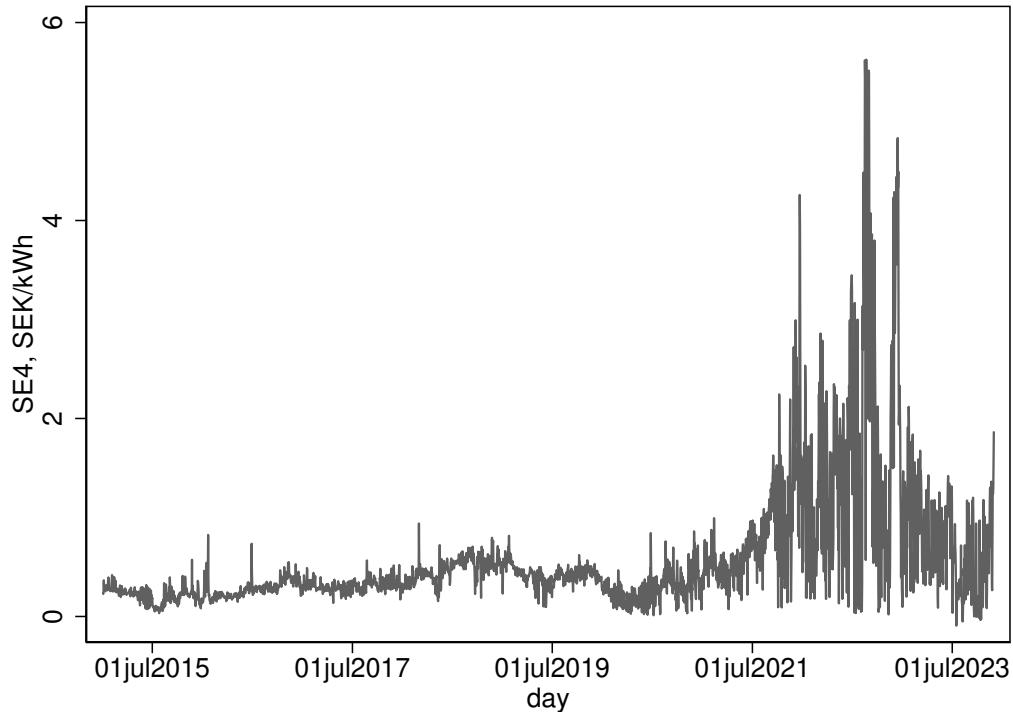


Figure 3: Daily average spot price in southern Sweden (SE4). Source: <https://www.nordpoolgroup.com/en/>

homes adequately warm according to figures for 2022, and this is an increase from previous years.² As already alluded to, while Sweden historically has had a relatively low prevalence of energy poverty, many households are unable to dodge price peaks and may need support in times of very high energy prices, such as during the last few years (von Platten (2022b); Antunes et al. (2023)). Increasing inflation on other goods and services has exacerbated the situation.

Energy price shocks have not only affect households, but also industry, and it seems likely that especially energy-intensive sectors are severely affected (e.g., UN (2022)). Most notably, such energy-intensive industries play an important role in many countries, with, for example, the iron and steel, minerals, refineries, and chemical industries combined employed an estimated 3.2 million people in the EU in 2019, accounting for approximately 11 percent

²See https://energy.ec.europa.eu/topics/markets-and-consumers/energy-consumer-rights/energy-poverty_en

of total industrial employment (UN (2022)). The fact that many of these industries produce intermediate goods, such as wood, paper and steel, implies that the energy price shocks transmit to the rest of the economies, and may have a wider impact on industrial output and competitiveness.

To mitigate the effects of high energy prices, policy makers in Europe and Sweden have suggested a range of policies, of which some also have been implemented. For example, the EU Commission has published the communication "Tackling rising energy prices: a toolbox for action and support" (see EU-commission (2022b)), which includes a range of measures, such as price caps, temporary tax breaks and social payments for households at the risk of energy poverty, and different type of energy vouchers and subsidies for consumers and firms. In the medium to long run, the suggested policies also include support mechanisms for expansion of supply and distribution. For example, EU-commission (2022b) mentions investments subsidies in renewable energy as a medium-run tool to mitigate the effects of high energy prices in the future, with the motivation that with more renewable energy in the energy market, the most expensive fossil fuels will be crowded out of the market, which reduces energy prices in the future.

Some of these measures have already been implemented: at the EU level (see EU-commission (2022a)), member states have agreed to reduce electricity consumption by 10 percent, and to reduce consumption in peak hours by 5 percent. Furthermore, most of the member states have implemented a temporary revenue cap on electricity production, where excess revenues³ from electricity production are redistributed to electricity customers.

Other measures have also been implemented on a national level. In Sweden, the government has implemented monetary transfers to all households in early 2022, to households in southern Sweden in early 2023, and again a subsidy to all households in mid 2023. In all these three cases, the value of the voucher was based on past consumption. To illus-

³Excess revenues is defined as prices about a reference price of €180/MWh

trate, the first voucher paid between 10 and 200 Euro depending on past consumption; the second voucher paid approximately 0.05 Euro per kWh of consumption between October 2021 and September 2022; the third voucher paid approximately 0.05 to 0.1 Euro per kWh of consumption in November and December 2022. See <https://www.svk.se/en/electricity-support-electricity-consumers/> for further details. Swedish firms also received monetary support, and in total, the supports in Sweden during 2022 and 2023 amounted to approximately 6 billion Euro. For the whole of Europe, the total value of support mechanisms since 2021 is approximately 650 billion euros.

In addition to the measures suggested by [EU-commission \(2022b\)](#) and implemented by member states such as Sweden (e.g., energy vouchers to households and firms), the recent energy price hikes have also sparked more interest in expanding domestic energy production. While the expansion of renewable energy has been advocated for long by governments and EU as key to reducing carbon emissions and enabling electrification of, e.g., transports and industries, the recent energy crisis has emphasized an even greater need for more generation to also ensure low and stable prices. For example, both EU and Sweden have started to discuss an expansion of nuclear energy production, including different forms of subsidies, in addition to the expansion of renewable energy.⁴

3 Literature Review

The literature on the macroeconomic effects of energy supply shocks is large, and started of with the early literature on the effects of the oil crisis of the 70's and 80's (see, for example, [Hamilton \(1983\)](#) and [Hamilton \(1988\)](#) and more recently [Kilian \(2008\)](#); [Hamilton \(2008\)](#)). These papers mostly find that energy price shocks in the past were often followed by large economic recessions, which suggests a causal link from higher energy prices to reductions in

⁴See, for example, [Halkos and Zisiadou \(2023\)](#) but also <https://www.euronews.com/my-europe/2023/09/26/european-commission-is-willing-to-consider-subsidies-for-nuclear-technology-says-von-der-l> and <https://www.regeringen.se/pressmeddelanden/2023/12/finansiering-och-riskdelning-vid-investeringar-i-nya-karnkraftsreaktorer/>

output, increasing unemployment, and higher inflation.

However, there is less consensus on the underlying mechanisms of the relation between energy crisis and output reductions, and this has implications for policies to mitigate the consequences of such energy price shocks. For example, [Kim and Loungani \(1992\)](#) use a dynamic stochastic general equilibrium model where energy is an input to production (but is not consumed directly by households), and show that energy price shocks can only explain a small part of the output fluctuations observed in U.S. data. On the other hand, [Hamilton \(2008\)](#) argues that oil shocks can significantly affect the economy by reducing spending on goods other than energy. For example, oil shocks can make consumers postpone their purchase of durable goods and in general reduce investments.

On a similar note, [Kilian \(2008\)](#) finds that a large increase in energy prices also leads a reduction in the demand for goods and services, and show that there may be several reasons for this: first, the negative income effect lowers consumers' purchasing power; second, the uncertainty effect that leads to consumers increasing their precautionary savings and reducing contemporaneous consumption; and third, some goods may be complementary to energy, and the demand for these goods fall as energy prices increase.

[Dhawan and Jeske \(2008\)](#) formulate a model that distinguishes between investment in consumer durables and capital goods, and, contrary to [Kim and Loungani \(1992\)](#), also includes energy use by households. This model is subsequently used to evaluate the consequences of energy price shocks on output fluctuations, and the results indicate that the effects of energy price shocks on output are to some extent mitigated by reducing investment in durables and in fixed capital.

[Cashin et al. \(2014\)](#) study the impacts of macroeconomic effects of both supply and demand shocks to the energy sector across a wide range of countries and macroeconomic variables, and show that the economic consequences of an energy shock depends on whether the shock is driven by increased demand or decreased supply. Furthermore, they show that while oil-

importing countries typically face a relatively long-term decrease in output in response to a supply-driven increase in oil prices, the impact is positive for energy-exporting countries, and especially those that possess large oil and gas reserves.

[van de Ven and Fouquet \(2017\)](#) analyze whether an economy's vulnerability and resilience to shocks depends on the level of economic development. Using 100 years of data on the United Kingdom, the paper distinguishes between supply and demand shocks to energy prices, and estimates how these shocks affect output. The results reveal that the impacts of supply shocks increased with the UK's increasing dependence on coal, and declined with the country's transition to oil.

[Balke and Brown \(2018\)](#) provide a model of the U.S. economy where energy is used both in production and in transportation services, where, in particular, the addition of a transportation sector allows the model to capture an important channel through which energy prices can affect economic activity. The model is a standard macroeconomic DSGE framework that includes nominal and real frictions, and using this model, the authors show that decreasing steady-state U.S. energy consumption substantially reduces the response of output to energy prices.

[Cai et al. \(2022\)](#) analyze OPEC and non-OPEC oil supply reductions and the effects on the euro area. They use a structural vector autoregressive regression model, and show that while both types of shocks decrease industrial output and increase unemployment, there is a difference between OPEC and non-OPEC energy supply shocks regarding the effects on consumer prices.

More recently, several working papers (e.g., [Lorenzoni and Werning \(2023\)](#); [Blanchard and Bernanke \(2023\)](#); [Gagliardone and Gertler \(2023\)](#)) all show that the last years' energy price shocks can explain recent inflation developments.

There are, as far as we are aware, few papers on energy shocks for the Nordic countries, but one exception is [Amundsen and Bergman \(2006\)](#) who investigate causes to why the supply

shock in 2002–2003 which occurred as a result of unusually low hydropower reservoir levels did not affect the Nordic countries to any large extent. Their analysis reveal that most of the negative effects of this supply shock was offset by increasing imports of energy, most notably from Russia and Germany.

Related is also [von Platten \(2022b\)](#), who explores vulnerability to heating-related energy poverty in Swedish single-family housing by analysing factors influencing households' ability to pay for heating and invest in energy efficiency. The results reveal that there are geographic, as well as socio-demographic factors, influencing the energy vulnerability experienced by Swedish households and that energy poverty in Sweden is accentuated in times of very high energy prices. See also [von Platten \(2022a\)](#).

Several studies have analyzed the effects of different support schemes to mitigate the consequences of energy supply shocks. For example, [Plante \(2014\)](#) constructs a model where both households and firms use energy, and show that in the long run, distortions from energy subsidies create welfare losses, and that the subsidies lead to crowding out of non-oil consumption, inefficient allocations of labor, and distortions in relative prices.

[Yau and Chen \(2021\)](#) measure the welfare effects of energy subsidies for an energy-importing country using a structural macroeconomic model that includes energy and durable goods consumption. The model is calibrated to fit the economy of Taiwan. Because Taiwan imports most of its energy, their model does not include energy production, and the supply of energy is exogenously given. The results show that subsidies to firms are better from a welfare perspective, compared to subsidies to households, and the reason for this result is that energy is mostly consumed by industries rather than by households. The welfare effects are, however, very small.

[Wildauer et al. \(2023\)](#) analyze the distributional effects of energy price shocks using a three-sector model (energy, goods and services). The model is calibrated to US data, and is used to analyse how energy shocks lead to redistribution of income between workers and firms

and between sectors of the economy. In brief, their results show that the shocks makes non-energy firms increasing their prices, which reduces real wages, and redistributes income towards energy firms. Finally, they compare three policies for mitigating the consequences of the energy price shock: redistributing windfall profits to workers, regulation on wages, and aggregate demand contraction through monetary or fiscal policy. They show that a redistribution of profits via a windfall tax is the most effective policy when it comes to reducing inflation without increasing unemployment.

[Pieroni \(2023\)](#) study the macroeconomic effects of energy supply shocks on European economies using a heterogeneous agents new Keynesian model with exogenous energy supply, and subsequently use the model to understand how fiscal and monetary policy can be used to mitigate the effects of supply shocks. The main findings is that income inequality (before the shock) amplifies the consequences of the energy shock, but that these consequences can be mitigated by monetary and fiscal policy.

During the last year, several working papers have analyzed the consequences of the recent energy price hikes, with a particular focus on the effects of different fiscal support mechanisms.⁵ For example, [Bayer et al. \(2023\)](#) study the effectiveness of fiscal responses to energy price shocks in a two-country heterogeneous agents new Keynesian model, calibrated to the German and Italian economies. A key feature in their model is that the total supply of energy in the union is inelastic, reflecting restrictions on energy import capacity. Using this modeling framework, they show that while energy subsidies can stabilize the domestic economy, they generate negative spillovers to the other country by increasing prices. Transfers based on historical energy consumption are less effective in the domestic economy than subsidies, but do not reduce economic activity abroad.

[Auclert et al. \(2023\)](#) is another recent working paper, and similarly to [Langot et al. \(2023\)](#); [Bayer et al. \(2023\)](#), use a heterogeneous-agent New Keynesian model to study the macroe-

⁵A number of papers have also studied macroeconomic policies in response to energy supply shocks; see, for example, [Chan et al. \(2022\)](#); [Pieroni \(2023\)](#).

conomic effects of energy price shocks in energy-importing economies. They show that increases in energy prices reduce income and leads to a recession, and while fiscal policy, such as energy subsidies, can mitigate the domestic effects of the shock, the policy has large negative externalities on other economies, which is in line with the results in [Bayer et al. \(2023\)](#).

Another recent working paper is [Langot et al. \(2023\)](#) who analyze the macroeconomic and re-distributive effects of a subsidy on energy. To that affect, they formulate a new-Keynesian business cycle model with heterogeneous agents (similar to [Bayer et al. \(2023\)](#)), and show that the energy subsidy leads to an increased growth, reduction in inflation and that it to some extent offset the increase in consumption inequality induced by the energy shock (the latter is the case even if the policy is not targeted at the poorest households).

Finally, the current paper focuses on energy supply shocks and how different policy instruments, suggested by policy makers, can mitigate the consequences of such shocks in the short to medium term. In the longer run, the key policy challenge is the green transition of societies and energy markets, with phasing out of fossil energy in favour of renewables. While this development may affect energy prices, and while households and firms may need supporting mechanisms during this period of transition, the analysis of this long-term development is outside the scope of the current paper (but see, for example, [Papież et al. \(2021\)](#); [Wang et al. \(2021\)](#); [Afshan et al. \(2024\)](#); [Gonzalez-Torres et al. \(2023\)](#)).

3.1 Our contribution

The current paper contributes to the previous literature reviewed above in several ways: first, the current paper studies the Swedish context, which is different from many other European countries in several important dimensions. For example, Sweden is very energy intensive (top ten in the world), which is mainly explained by long and cold winters and an energy-intensive industry. Furthermore, Sweden has a lot of domestic energy production (especially electricity production) and a low import dependence on energy, whereas many

other European countries rely to a large extent on imports of energy (e.g., Germany) and electricity (e.g., Italy and France).⁶

Second, and related to the first item, while several papers have analyzed the macroeconomic effects of energy shocks and how fiscal policy can mitigate these effects, many of them do not model the energy sector, but rather assume that energy supply is exogenous (e.g., Bayer et al. (2023); Langot et al. (2023); Pieroni (2023)). One motivation for this may be that the domestic energy production in many countries is small, and that many countries depend to a large extent on imports of energy. The current paper, on the other hand, explicitly models the energy supply. Importantly, this allows us to investigate not only the effect of demand-side policies, but also the effects of support schemes targeting the production of energy, such as investment subsidies to energy-producing firms. Measures to accelerate the expansion of renewable energy have been highlighted by EU-commission (2022b) as a way to mitigate the effect of high, and our model allows us to understand how such policies compare to, e.g., energy vouchers to households and firms. Obviously, such policies are only relevant for economies with domestic energy production, which further motivates our focus on Sweden.

Third, we contribute to the literature on supply-side energy policies aiming at increasing supply (most notably the supply of renewables); see, for example, Böhringer et al. (2022); Trujillo-Baute et al. (2018); Schmalensee (2012). Specifically, we explore how an investment subsidy targeting the production of energy affects a range of macroeconomic quantities, including energy prices and consumption.

Fourth, we allow for heterogeneity among households across two dimensions: their skill level, and whether they are rational savers or live hand-to-mouth. This is different from many previous papers, such as Yau and Chen (2021); Wildauer et al. (2023), that model the

⁶See, for example, <https://www.statista.com/statistics/1405405/net-electricity-exports-europe-by-country/> and https://ec.europa.eu/eurostat/statistics-explained/index.php?title=Archive:EU_energy_mix_and_import_dependency.

behavior of a representative household, but is in line with an emerging strand of literature exploring the effects of energy supply shocks when households are heterogeneous (e.g., [Auclert et al. \(2023\)](#); [Pieroni \(2023\)](#); [Chan et al. \(2022\)](#)). In particular, this heterogeneity allows us to analyze re-distributive effects of both energy shocks and associated policies, and also allow us to explore the effects of policies targeting low-income households and households that are unable to mitigate the effects of price shocks, even if they are relatively short term. This aspect of energy poverty is highlighted in recent literature; see, for example, [von Platten \(2022b\)](#) for the Swedish context.

To summarize, the current paper makes several important contributions to the existing literature on the consequences of energy shocks to the economy. Next, we turn to a more detailed description of our modeling framework.

4 Model

Time is discrete and denoted t . Consider an economy that consists of heterogeneous households, differing in productivity and optimization behavior, an energy sector, a non-energy sector that produces non-durable goods, and a government. The energy good is both consumed by households and used as a factor input good in the production of non-energy goods. The government collects revenue from labor income taxation, and uses this revenue to finance different fiscal policies. The fiscal policy mix we consider are: an energy voucher to all households, an energy voucher to low-income households, an energy voucher to non-energy producers, and a subsidy for capital investments in the energy sector.

4.1 Energy Producer

Let the energy producer be denoted by index e . We assume that the energy producer only uses capital K_e as input in the production function:⁷

⁷Data from Statistics Sweden shows that the expenditure share on capital in the sector for heating and electricity is 0.63. The average for all other sectors is 0.16. Regarding labor, the share is 0.35 for heating

$$Y_{e,t} = A_{e,t}K_{e,t-1}, \quad (1)$$

where $Y_{e,t}$ is the output. The technology $A_{e,t}$ is specific to the energy sector and is assumed to follow a stochastic process:

$$\log(A_{e,t}) = (1 - \rho_{A_e}) \log(A_e) + \rho_{A_e} \log(A_{e,t-1}) + \varepsilon_{A_e,t} \quad (2)$$

where $\varepsilon_{A_e,t}$ is the technology shock, and ρ_{A_e} is an autoregressive parameter that captures the persistence of the shock. We require this process to be stationary, which implies that $|\rho_{A_e}| < 1$.

The energy producer's profit writes:

$$\Pi_{e,t} = Y_{e,t} - R_{e,t}K_{e,t-1}, \quad (3)$$

and we assume that the energy producer earns zero-profit:

$$R_{e,t} = P_{e,t}A_{e,t}, \quad (4)$$

which defines the capital demand of the energy producer. This implies that the real rate of return is equal to the value of the marginal product of capital.

4.2 Non-energy Goods Producer

Let the non-energy producer be denoted by index n . The non-energy goods producer uses a composite of labor H , capital K_n , and energy E_n to produce a single non-durable good. We assume a CES production function:

$$Y_{n,t} = A_{n,t} \left[\mu_{nk}^{\frac{1}{\varepsilon_n}} K_{n,t}^{\frac{\varepsilon_n-1}{\varepsilon_n}} + \mu_{ne}^{\frac{1}{\varepsilon_n}} \left(E_{n,t} + \frac{T_{n,e,t}}{P_{e,t}} \right)^{\frac{\varepsilon_n-1}{\varepsilon_n}} + (1 - \mu_{nk} - \mu_{ne})^{\frac{1}{\varepsilon_n}} H_t^{\frac{\varepsilon_n-1}{\varepsilon_n}} \right]^{\frac{\varepsilon_n}{\varepsilon_n-1}}, \quad (5)$$

and electricity, while it is on average 0.78 for all other sectors.

where $\mu_{nk}, \mu_{ne} \in (0, 1)$ are the shares of capital and energy used as inputs in the production of non-energy goods. ε_n is the elasticity of substitution between the inputs. $T_{n,e,t}$ is an energy voucher provided by the government for energy consumption only.

The technology $A_{n,t}$ is specific to the non-energy goods sector and is assumed to follow a stochastic process:

$$\log(A_{n,t}) = (1 - \rho_{A_n}) \log(A_n) + \rho_{A_n} \log(A_{n,t-1}) + \varepsilon_{A_{n,t}} \quad (6)$$

where $\varepsilon_{A_{n,t}}$ is the technology shock, and ρ_{A_n} is an autoregressive parameter which captures the decay of the shock. We assume also this process to be stationary, which implies that $|\rho_{A_n}| < 1$.

Normalizing the price of the non-energy good to unity, the non-energy goods producer maximizes profit by solving the following problem:

$$\max_{K_{n,t}, H_t, E_{n,t}} \Pi_{n,t} = Y_{n,t} - W_{n,t} H_t - R_{n,t} K_{n,t-1} - P_{e,t} E_{n,t}. \quad (7)$$

The first order conditions for the maximization problem are:

$$R_{n,t} = \mu_{nk}^{\frac{1}{\varepsilon_n}} A_{n,t} \left(\frac{Y_{n,t}}{A_{n,t} K_{n,t-1}} \right)^{\frac{1}{\varepsilon_n}}, \quad (8)$$

$$P_{e,t} = \mu_{ne}^{\frac{1}{\varepsilon_n}} A_{n,t} \left(\frac{Y_{n,t}}{A_{n,t} E_{n,t}^*} \right)^{\frac{1}{\varepsilon_n}}, \quad (9)$$

$$W_t = (1 - \mu_{nk} - \mu_{ne})^{\frac{1}{\varepsilon_n}} A_{n,t} \left(\frac{Y_{n,t}}{A_{n,t} H_t} \right)^{\frac{1}{\varepsilon_n}}. \quad (10)$$

where $E_{n,t}^* = E_{n,t} + \frac{T_{n,e,t}}{P_{e,t}}$. That is, the firm decides on the level of capital, energy, and labor, such that the value of the marginal product of the input is equal to its factor price.

4.3 Households

Households either admit to Ricardian equivalence and save rationally, or live hand-to-mouth and do not save by assumption. This heterogeneity is included to account for the stylized fact that some households do not save (see, e.g., von Platten (2022b)). Let superscript $j = RS$ denote Ricardian savers and $j = HTM$ denote hand-to-mouth households. Except when explicitly needed, this index will be suppressed to avoid notational clutter. Ricardian households also differ in skill, indexed by i , where $i = 1$ denotes low-skilled households and $i = 2$ is high-skill. We further assume that all hand-to-mouth households are low-skilled. Normalizing the population to 1, let Λ_i^{RS} and Λ^{HTM} denote the corresponding population weights such that $\Lambda^{HTM} + \sum_{i=1}^2 \Lambda_i^{RS} = 1$.

At each instant in time, the Ricardian households derive utility from total consumption $c_{i,t}^{RS}$ and leisure $1 - h_{i,t}^j$, where $h_{i,t}^j$ is their labor supply. Assuming additive separable CRRA preferences, each household type maximizes the expected sum of its intertemporal utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(c_{i,t}^j)^{1-\gamma}}{1-\gamma} - \varkappa \frac{(h_{i,t}^j)^{1+\sigma}}{1+\sigma} \right), \quad (11)$$

where $\gamma > 0$ is the inverse of the elasticity of intertemporal substitution of consumption and $\sigma > 0$ denotes the inverse of the Frisch elasticity of labor supply. $\varkappa > 0$ is the weight attached to the disutility of labor. The total consumption is a composite of consumption of the energy $c_{e,i,t}^j$ and non-energy goods $c_{n,i,t}^j$, defined by a CES integration:

$$c_{i,t}^j = \left(\mu_e^{\frac{1}{\epsilon}} \left(c_{e,i,t}^j + \frac{T_{h,e,i,t}}{P_{e,t}} \right)^{\frac{\epsilon-1}{\epsilon}} + (1 - \mu_e)^{\frac{1}{\epsilon}} (c_{n,i,t}^j)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \quad (12)$$

where μ_e is the proportion of energy in the composite good in steady state, and ϵ the elasticity of substitution between the different types of goods. The household also receives an energy voucher $T_{h,e,i,t}$ which is a lump-sum transfer that is earmarked for the consumption of energy.

Since the energy sector only demands capital as input, total hours worked by the household $h_{i,t}^j$ is in the non-energy good sector only:

$$h_{i,t}^j = h_{n,i,t}^j. \quad (13)$$

A Ricardian household's budget constraint is given by:

$$\begin{aligned} c_{n,i,t}^{RS} + P_{e,t}c_{e,i,t}^{RS} + inv_{n,i,t}^{RS} + (1 - s_{e,t})inv_{e,i,t}^{RS} + \frac{\psi_n}{2}(k_{n,i,t}^{RS} - k_{n,i,t-1}^{RS})^2 + \frac{\psi_e}{2}(k_{e,i,t}^{RS} - k_{e,i,t-1}^{RS})^2 \\ \leq (1 - \varpi_i)W_t\epsilon_i h_{i,t}^{RS} + T_{i,t} + R_{n,t}k_{n,i,t-1}^{RS} + R_{e,t}k_{e,i,t-1}^{RS} \end{aligned} \quad (14)$$

where $P_{e,t}$ is the relative price of energy. ϖ is the income tax rate. Since households are the owner of the capital stock, and we abstract from financial intermediaries, they carry the investment adjustment costs $\frac{\psi_n}{2}(k_{n,i,t}^{RS} - k_{n,i,t-1}^{RS})^2$ and $\frac{\psi_e}{2}(k_{e,i,t}^{RS} - k_{e,i,t-1}^{RS})^2$. $s_{e,t}$ is the subsidy households receive when they invest in the energy sector. For each unit of labor supplied, the households earn an effective wage rate of $W_t\epsilon_i$, where W_t is the per-unit of efficient labor wage rate, and ϵ_i reflects the households marginal productivity. $T_{i,t}$ is a lump-sum transfer which can differ between skill-groups, but is independent of whether the household saves rationally or lives hand-to-mouth. $inv_{j,i,t}^{RS}, j = \{n, e\}$ is the investment in energy and non-energy sectors, which is defined as :

$$inv_{j,i,t}^{RS} = k_{j,i,t}^{RS} - (1 - \delta_j)k_{j,i,t-1}^{RS}, j = \{n, e\} \quad (15)$$

The Ricardian households choose $\{c_{e,i,t}^{RS}, c_{n,i,t}^{RS}, h_{i,t}^{RS}, k_{n,i,t}^{RS}, k_{e,i,t}^{RS}\}$ to maximize their lifetime utility function subject to the budget constraint. The first order conditions are:

$$\lambda_{i,t}^{RS} = (1 - \mu_e)^{\frac{1}{\epsilon}} (c_{i,t}^{RS})^{-\gamma + \frac{1}{\epsilon}} (c_{n,i,t}^{RS})^{-\frac{1}{\epsilon}} \quad (16)$$

$$P_{e,t} \lambda_{i,t}^{RS} = \mu_e^{\frac{1}{\epsilon}} (c_{i,t}^{RS})^{-\gamma + \frac{1}{\epsilon}} (c_{e,i,t}^{RS} + \frac{T_{h,e,i,t}}{P_{e,t}})^{-\frac{1}{\epsilon}} \quad (17)$$

$$(1 - \varpi_i) \epsilon_i W_t \lambda_{i,t}^{RS} = \varkappa (h_{i,t}^{RS})^\sigma \quad (18)$$

$$\lambda_{i,t}^{RS} = E_t \beta \frac{R_{n,t+1} + 1 - \delta_n + \psi_n (k_{n,i,t+1}^{RS} - k_{n,i,t}^{RS})}{1 + \psi_n (k_{n,i,t}^{RS} - k_{n,i,t-1}^{RS})} \lambda_{i,t+1}^{RS} \quad (19)$$

$$\lambda_{i,t}^{RS} = E_t \beta \frac{R_{e,t+1} + (1 - \delta_e)(1 - s_{e,t+1}) + \psi_e (k_{e,i,t+1}^{RS} - k_{e,i,t}^{RS})}{(1 - s_{e,t}) + \psi_e (k_{e,i,t}^{RS} - k_{e,i,t-1}^{RS})} \lambda_{i,t+1}^{RS} \quad (20)$$

For the hand-to-mouth consumers, their utility function and consumption functions are similar to the rational saving households. The difference between hand-to-mouth and rational saving is that since the hand-to-mouth household type by construction does not save, their budget constraint collapses to a static trade-off:

$$c_{n,t}^{HTM} + P_{e,t} c_{e,t}^{HTM} \leq (1 - \varpi_1) W_t \epsilon_1 h_t^{HTM} + T_{1,t} \quad (21)$$

The first order conditions for hand-to-mouth households are:

$$\lambda_t^{HTM} = (1 - \mu_e)^{\frac{1}{\epsilon}} (c_t^{HTM})^{-\gamma + \frac{1}{\epsilon}} (c_{n,t}^{HTM})^{-\frac{1}{\epsilon}} \quad (22)$$

$$P_{e,t} \lambda_t^{HTM} = \mu_e^{\frac{1}{\epsilon}} (c_t^{HTM})^{-\gamma + \frac{1}{\epsilon}} (c_{e,t}^{HTM} + \frac{T_{1,t}}{P_{e,t}})^{-\frac{1}{\epsilon}} \quad (23)$$

$$(1 - \varpi_1) \epsilon_1 W_t \lambda_t^{HTM} = \varkappa (h_t^{HTM})^\sigma \quad (24)$$

4.4 Aggregation

The aggregates are the weighted sum of household quantities. For household consumption of the non-energy good:

$$C_{n,t} = \Lambda^{HTM} c_{n,t}^{HTM} + \sum_{i=1}^2 \Lambda_i^{RS} c_{n,i,t}^{RS} \quad (25)$$

household consumption of the energy-good:

$$C_{e,t} = \Lambda^{HTM} \left(c_{e,t}^{HTM} + \frac{T_{h,e,1,t}}{P_{e,t}} \right) + \sum_{i=1}^2 \Lambda_i^{RS} \left(c_{e,i,t}^{RS} + \frac{T_{h,e,i,t}}{P_{e,t}} \right), \quad (26)$$

labor supply:

$$H_t = \Lambda^{HTM} \epsilon_1 h_t^{HTM} + \sum_{i=1}^2 \Lambda_i^{RS} \epsilon_i h_{i,t}^{RS}, \quad (27)$$

and the capital supply for the production of the non-energy good:

$$K_{n,t} = \sum_{i=1}^2 \Lambda_i^{RS} k_{n,i,t}^{RS}, \quad (28)$$

the production of energy:

$$K_{e,t} = \sum_{i=1}^2 \Lambda_i^{RS} k_{e,i,t}^{RS}, \quad (29)$$

and investment :

$$INV_{e,t} = \sum_{i=1}^2 \Lambda_i^{RS} inv_{e,i,t}^{RS}, \quad (30)$$

$$INV_{n,t} = \sum_{i=1}^2 \Lambda_i^{RS} inv_{n,i,t}^{RS} \quad (31)$$

4.5 Government

We assume that the government operates on a balanced budget at each instant in time. That is, we rule out the option of debt-financing fiscal policy. Revenue is collected from labor income taxes such that $\varpi W_t H_t$ is the aggregate revenue. The budget constraint of the public sector is given by:

$$T_t + T_{h,e,t} + T_{n,e,t} + s_{e,t} K_{e,t} = \varpi W_t H_t \quad (32)$$

That is, any revenue not used for financing the energy vouchers or the investment subsidy goes towards a lump-sum transfer to the households. Both the lump-sum transfers and the

energy voucher could be designed to vary with the skill-level of the household:

$$T_t = (\Lambda^{HTM} + \Lambda_1^{RS})T_{1,t} + \Lambda_2^{RS}T_{2,t}, \quad (33)$$

and

$$T_{h,e,t} = (\Lambda^{HTM} + \Lambda_1^{RS})T_{h,e,1,t} + \Lambda_2^{RS}T_{h,e,2,t}. \quad (34)$$

We assume that the lump-sum transfer to low-income households is proportional to that to high-income households:

$$T_{1,t} = \alpha T_{2,t}. \quad (35)$$

4.6 Market Clearing Conditions

Based on the resource constraint condition⁸:

$$\begin{aligned} P_{e,t}Y_{e,t} + Y_{n,t} &= P_{e,t}E_{n,t} + P_{e,t}C_{e,t} + T_{he,t} + P_{e,t} + C_{n,t} + INV_{n,t} + INV_{e,t} \\ &+ \sum_{i=1,2} \Lambda_i^{RS} \frac{\psi_n}{2} (k_{n,i,t}^{RS} - k_{n,i,t-1}^{RS})^2 + \sum_{i=1,2} \Lambda_i^{RS} \frac{\psi_e}{2} (k_{e,i,t}^{RS} - k_{e,i,t-1}^{RS})^2 \end{aligned} \quad (36)$$

We assume that all capital in the economy is derived one-to-one from the non-energy good.

The capital adjustment costs are also measured by the units of non-energy goods.

Last, we impose the condition that the market of energy goods clears :

$$Y_{e,t} = E_{n,t} + C_{e,t} + \frac{T_{he,t}}{P_{e,t}} + \frac{T_{ne,t}}{P_{e,t}} \quad (37)$$

5 Calibration

We calibrate the model to match the steady state values for the real economic variables according to aggregate consumer and production statistics, based on recent Swedish data.

For the few parameters where we lack targets in the data, we resort to using standard values

⁸The derivation of resource constraint can be found in Appendix.

from the business cycle literature.

The Swedish context

Starting with the cost shares in the production of non-energy goods, we have used firm-level panel data from Sweden on cost shares, covering the years 2003-2021. The data is sourced from Statistics Sweden and covers all manufacturing industries in Sweden. Older versions of these data have been used in previous papers; see, for example, [Amjadi et al. \(2018\)](#); [Dahlqvist et al. \(2021\)](#). We subsequently set $\mu_{nk} = 0.18$ and $\mu_{ne} = 0.05$. For the capital depreciation rates, we follow [Edquist and Henrekson \(2017\)](#) and set them to 0.087. This is similar to standard values in the literature.

For the elasticity of substitution between energy and capital and labor in production, we follow [Broberg et al. \(2015\)](#) who develop a CGE model for Sweden, and set this parameter to 0.8. The capital adjustment cost parameters are set to $\phi_e = \phi_n = 10$, and we perform a sensitivity analysis concerning these parameters in Appendix C.

According to Statistics Sweden (see <https://www.statistikdatabasen.scb.se>), the average annual income in Sweden was approximately 32,000 euros in 2022. According to the same source, the average electricity expenditure for households was 1300 euros. This implies that on average, households spend approximately ten percent of their income on electricity. Thus, we set $\mu_c = 0.1$.

To compute the skill premium, we use annual earnings statistics for Sweden in 2015, including both natives and immigrants, as reported in [Friedrich et al. \(2021\)](#). Specifically, we calculate that $P75/P25=1.8$, which we match by first normalizing $\epsilon_1 = 1$ and then setting $\epsilon_2 = 1.8$. Regarding measures of the proportion of households living hand-to-mouth, a recent study

by von Platten (2022a) show that approximately 20 percent of the Swedish households often need to reduce their indoor heating to reduce costs. These results are based on a national survey in 2021. We interpret these results as 20 percent of households being financially constrained, and use these results as a basis for our parameter Λ^{HTM} , setting the share of hand-to-mouth households equal to 0.2 We assume that the rest of the population is divided between high- and low-skilled such that 50 percent of the population is high-income (i.e., $\Lambda_1^{RS} = 0.3$ and $\Lambda_2^{RS} = 0.5$).

The total labor income tax wedge in Sweden is currently 42.4 %. Consequently, we set $\varpi = 0.424$.⁹ We calculate the overall redistribution of the Swedish tax/transfer system by taking the difference between the Gini coefficient for factor income and the Gini coefficient for disposable income in per cent of the Gini coefficient for factor income (see OECD (2023)). This gives us a value of roughly 27 %, which we use to parameterize $\alpha = 1.27$.

To the best of our understanding, empirical evidence on the relevant Frisch elasticity (accounting for both intensive and extensive margin responses to a change in the wage rate) for Sweden is scant at best. Previous literature that calibrate DSGE models to Sweden have generally assumed the inverse of the Frisch elasticity σ to be around 2 (Olovsson, 2009).

\varkappa allows us to scale the fraction of time that the household devotes to work. It is standard to target a value of $H_t = 0.33$, and this corresponds well to the Swedish context.

General business cycle literature

The discount factor is set to $\beta = 0.97$ which corresponds to an annual discount rate close to 4%. The inverse elasticity of intertemporal substitution is set to $\gamma = 1$ which is consistent with balanced growth (Lucas Jr, 1990).

⁹see, e.g., <https://www.oecd.org/tax/tax-policy/taxing-wages-brochure.pdf>

Table 1: Calibration of structural parameters and steady state values

Description	Parameter	Value	Source
<i>The Swedish context</i>			
Share of capital used as an input in the production of non-energy goods	μ_{nk}	0.18	Statistics Sweden/own calculation
Share of electricity used as an input in the production of non-energy goods	μ_{ne}	0.05	Statistics Sweden/own calculation
Capital depreciation rate	δ_n, δ_e	0.087	Edquist and Henrekson (2017)
Elasticity of substitution between energy and capital and labor in production	ϵ_n	0.8	Broberg et al. (2015)
Share of energy consumption	μ_e	0.1	Statistics Sweden/own calculation
Elasticity of substitution between energy and non-energy goods consumption	ϵ	0.8	authors' calibration
Inverse of the wage elasticity of labor supply	σ	2	Olovsson (2009)
Productivity low-skilled	ϵ_1	1	Friedrich et al. (2021)
Productivity high-skilled	ϵ_2	1.8	Friedrich et al. (2021)
Share of hand-to-mouth	Λ^{HTM}	0.2	von Platten (2022b)
Share of low-income Ricardian households	Λ_1^{RS}	0.3	Statistics Sweden/own calculation
Share of high-income Ricardian households	Λ_2^{RS}	0.5	Statistics Sweden/own calculation
Income tax rate	ϖ	0.424	Tax wedge on labor income
Proportion of lump-sum to low-income households over high-income households	α	1.27	OECD data/own calculation
Dis-utility of work	\varkappa	40.2830	Works 1/3 of time endowment
<i>Business cycle literature</i>			
Discount factor	β	0.97	Annual discount rate of 3%
Inverse of the elasticity of intertemporal substitution of consumption	γ	1	Lucas Jr (1990)
<i>Other</i>			
Parameter for adjustment cost	ϕ_n	10	
Parameter for adjustment cost	ϕ_e	10	

6 Simulation

First, we simulate the potential effects of the different fiscal instruments: energy vouchers given to: all households, low-income households, and non-energy firms, and a subsidy for investments in the energy sector, in addition to a business-as-usual (BAU) scenario in which the government does not specifically address the negative energy supply shock. As it is complicated to model the international geopolitical situation, which is the cause of energy supply shock/crisis, therefore, in our model, for simplification, the energy supply shock is captured by a fall of productivity in the energy sector. Based on the policy debate (see Sections 1 and 2), we consider that the energy price is likely a key indicator of interest for the government. This experiment is contained in Section 6.1. Second, as evaluating policies

based on prices alone likely masks important welfare effects, we complete the analysis by comparing the welfare effects in Section 6.2.

6.1 Real economic effects

Figure 4 and 5 depict the real economic effects of a -1pp negative TFP shock, and the comparison with the application of 4 different fiscal policies, with the same budget as 1% of GDP.

Let us first consider the BAU scenario¹⁰, in which the government does not intervene in the market adjustment back to the pre-shock steady state. As energy is consumed both directly by the household $C_{e,t}$, and also used as an input in the production of non-energy goods $E_{n,t}$, the negative TFP shock to the energy sector implies a negative aggregate supply shock to the economy as a whole. As the shock reduces energy output $Y_{e,t}$ in the economy, the price of energy increases. The increased energy price, in turn, makes the production of non-energy goods more expensive, such that output falls also in this sector. This increases the price of the non-energy good as well, but relatively less compared to the energy price. Ultimately, the relative price of energy $P_{e,t}$ increases. The reduced output and higher energy price increases the value of the marginal product of capital in this sector (see Eq. 4), which implies an increase in the real rate of return of investments in capital for the energy sector. As such, investments in the energy sector increase which partially offsets the negative output gap. As the negative TFP shock decays, the economy gradually reverts back to steady-state.

Now, let us instead consider that the government intervenes through active fiscal policy. The investment subsidy (EV2INV) leads to increased capital investments in the energy sector $K_{e,t}$. As a result, energy output $Y_{e,t}$ increases such that energy prices fall more rapidly relative to the other policies and the BAU scenario.

¹⁰In Figure 4 and 5, IRF's associated with the baseline scenario is very close to the scenario with energy vouchers to households. In Appendix B, we illustrate the deviations from the BAU scenario.

As the relative price of energy falls, households consume relatively more energy goods so that $C_{e,t}^*$ increases, and relatively less non-energy goods, so that $C_{n,t}$ decreases. Given the reduced relative price, the demand for energy as an input good in the production of non-energy goods also increase. The initial drop in $C_{e,t}^*$ can be explained by the following mechanism: when non-energy goods consumption $c_{n,t}^j, j = \{rs, htm\}$ decreases, the marginal utility $\lambda_{i,t}^j, j = \{rs, htm\}$ increases according to equation 16 and 22. Consequently, from equation 17 and 23, in the short term, this effect on the energy consumption is dominant. Later, the effect from a relatively lower energy price becomes dominant and the energy consumption increases.

Recall that the investment/capital goods are assumed to be derived one to one from non-energy goods. As such, the increase in non-energy production $Y_{n,t}$ reflects an increase in savings in the energy sector, corresponding to an increase in $K_{e,t}$. This explains why the overall demand for non-energy goods increases despite of the fall of consumption of non-energy goods.

As the energy production increase, the marginal productivity of capital in this sector falls. This effect, coupled with the fall in the energy price, implies that the returns to investments in the energy sector $R_{e,t}$ also falls (see equation 4). Meanwhile, as the subsidy gives incentives for the household to divert investments from the non-energy sector to the energy sector, capital $K_{n,t}$ falls in this sector. As a result, the remaining capital stock is more productive, and consequently, returns to non-energy capital return, i.e., the interest rate $R_{n,t}$, increases. Concerning the labor market, as investment subsidy increases non-energy goods production, the return of labor increases (see equation 24). Households will therefore work more and there is more labor supply in the non-energy sector. In equilibrium, the labor supply effect is dominant and the wage level in equilibrium W_t falls.

Turning to the energy vouchers to households, we compare the outcomes for our indicators of giving the voucher only to low-income households (EV2LHH), in an egalitarian effort, or to all households (EV2HH).

In the scenario where energy vouchers are given to low-income households only, the household voucher policy effectively corresponds to a redistribution of funds from high- to low-income households. As a non-trivial fraction of low-income households live hand-to-mouth, this policy reduces total savings in the economy. The total effect is however found to be very small, but it does give rise to an increase in the returns to both non-energy capital and returns to energy capital. If, instead, the energy voucher is given to all households, the real economic outcome does not diverge much from the BAU scenario. Again, the negative TFP shock to the energy producer leads to lower energy output and increased energy prices. As a result, capital becomes more productive in this sector, and the value of the marginal product increase following an increase in the energy price. Consequently, the rate of return to investments in the energy sector increases, and households redirect some of their investments from the non-energy sector towards savings in the energy sector. This contributes to increasing the energy supply, and subsequently, the energy price falls. Overall, the real economic effects of giving energy vouchers to the households are found to be very small relative to the BAU scenario.

As for the energy vouchers directly given to non-energy firms (EV2F), the voucher leads to an increase in the non-energy firms' demand for energy as input in the production. As non-energy firms consume more energy, the rise of demand drives up the energy price $P_{e,t}$.¹¹ As the energy goods become more expensive, the total consumption of energy goods $C_{e,t}^*$ falls. As energy goods and non-energy goods are complementary for households, households also consume less non-energy goods. The total energy input for non-energy firms $E_{n,t}^*$ first

¹¹In the very short term, the energy price $P_{e,t}$ decreases, as shown in the net effects of fiscal instruments Figures 8 and 9. This short-term effect originates from equation 9. However, in the medium and long run, the demand effects become dominant and the energy price $P_{e,t}$ rises.

rises then declines due to the increase in the relative price. The total energy production $Y_{e,t}$ falls. The wage level increases in very short term according to equation 10; this encourages households to work more, and the total working hours H_t increase. In general equilibrium, the labor supply effect is dominant, and the wage level declines. Because of the increase of labor input, the production of non-energy goods $Y_{n,t}$ increases. As the wage level decreases, the investment of households in both the energy and non-energy sectors fall, and capital $K_{e,t}$ and $K_{n,t}$ decline. The interest rate in the non-energy sector $R_{n,t}$ decreases because of the fall of capital from households. The interest rate in the energy sector $R_{e,t}$ first falls because of the fall of the energy price (equation 4). In the medium and long run, it increases because of the rise of energy prices and the fall of investments from households.

Ultimately, from this analysis, we conclude that if the primary aim of the government is to reduce energy prices at a faster rate compared to the reduction achieved through the market adjustment, the subsidy for capital investments in the energy sector is the preferable policy option of the menu considered here.

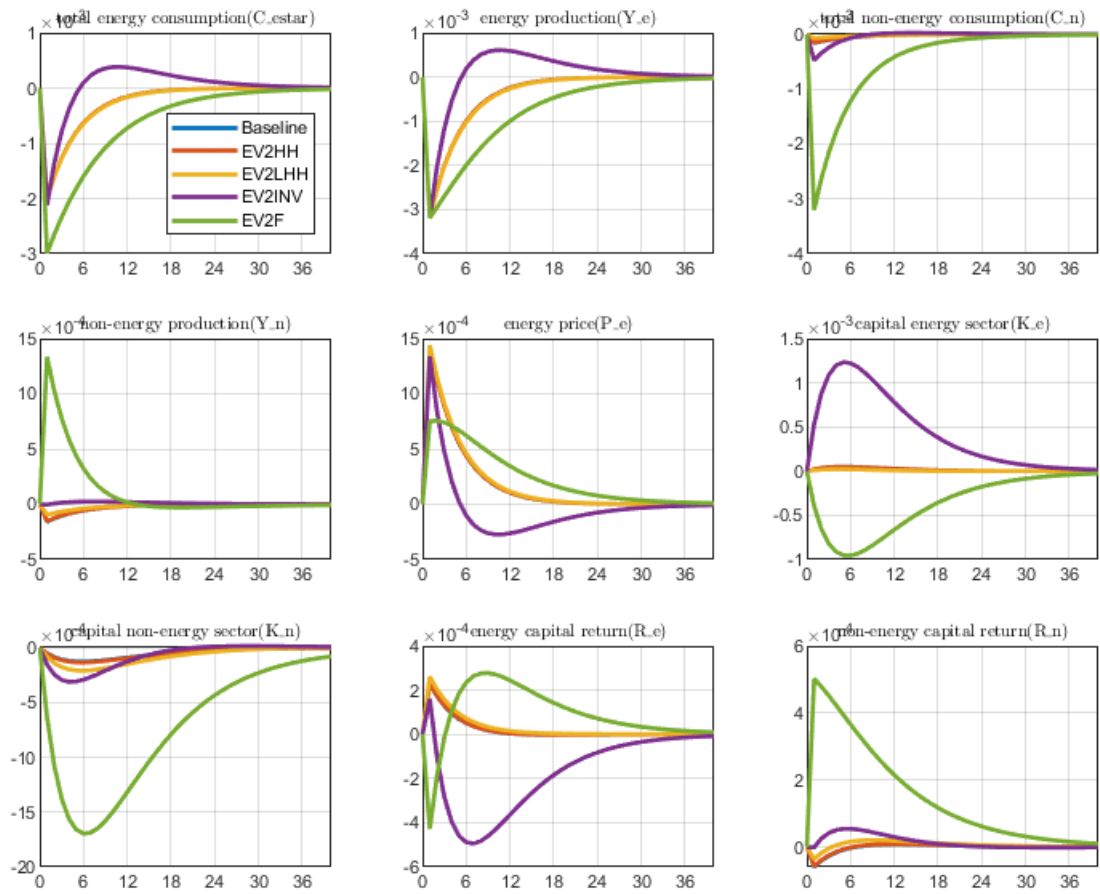


Figure 4: Effects of 4 fiscal instruments on the baseline scenario (Business-As-Usual), with the same budget as 1% of GDP - 1

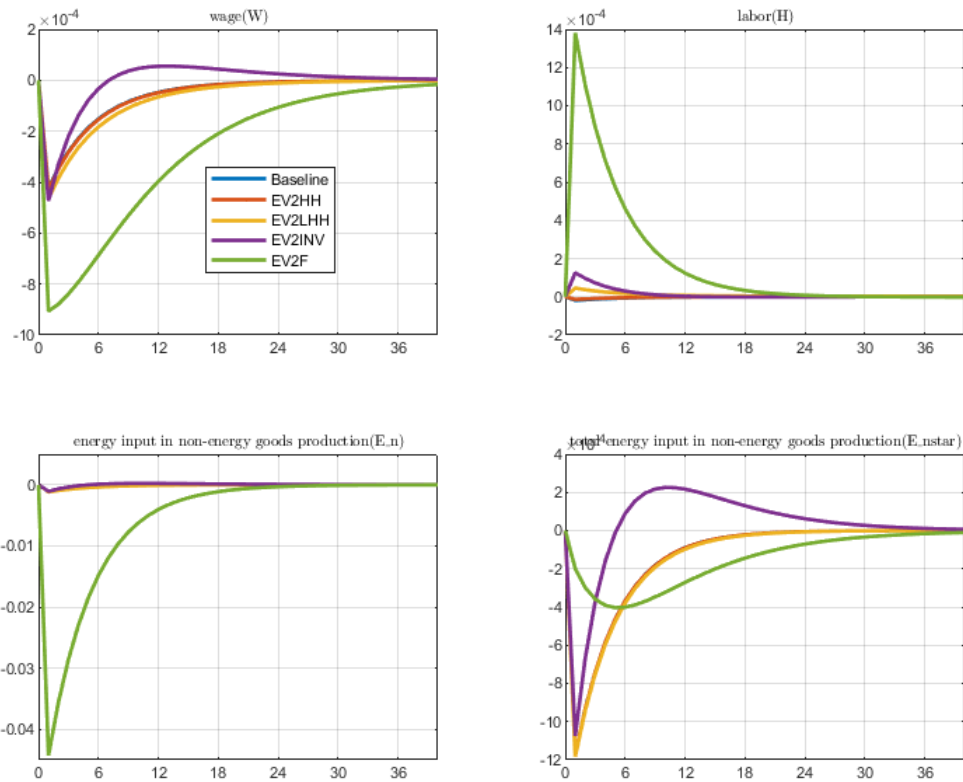


Figure 5: Effects of 4 fiscal instruments on the baseline scenario (Business-As-Usual), with the same budget as 1% of GDP - 2

6.2 Welfare Analysis

So far, we have studied the effects of different fiscal policies on a rich set of real economic variables. While policy makers, and their voters for that matter, could be interested in targeting one particular economic variable, such as the energy price, this likely masks important welfare effects. To complete the analysis, we therefore simulate the welfare effects of the different fiscal instruments. The welfare effects are computed relative to a baseline scenario in which there is -1pp shock on the productivity of the energy sector $A_{e,t}$.

Welfare WF_t is calculated by using the following recursive function:

$$WF_t = UT_t + \beta WF_{t+1}$$

where UT_t is the weighted sum of household utilities:

$$UT_t = \Lambda_1^{RS} ut_{1,t}^{RS} + \Lambda_2^{RS} ut_{2,t}^{RS} + \Lambda^{HTM} ut_t^{HTM} \quad (38)$$

$$ut_{i,t}^j = \frac{(c_{i,t}^j)^{1-\gamma}}{1-\gamma} - \varkappa \frac{(h_{i,t}^j)^{1+\sigma}}{1+\sigma}, j = \{RS, HTM\}, i = \{1, 2 \text{ if } j = RS\} \quad (39)$$

The welfare effects are presented in Table 3, where we have sorted the policies from the smallest welfare effect to the largest.

A somewhat surprising result is that the subsidy for capital investments in the energy sector is welfare-dominated by energy vouchers to households. The intuition to this result is that this subsidy only directly benefits rational savers of both high- and low-income. Indeed, subsidizing investments does not compensate the hand-to-mouth households for their lack of insuring measures against potential energy poverty. As such, it does not *directly* reduce the burden on hand-to-mouth consumers of the energy supply shock. Also, relative to the BAU scenario, the hand-to-mouth households now receive a lower lump-sum transfer. Any positive effect for hand-to-mouth households instead follows indirectly through a swifter reduction in energy prices.

The energy voucher given to firms results in lower consumption of all goods, and less leisure, when compared to the BAU scenario. As a result, this policy results in the lowest level of welfare.

To summarize, our results show that if the government aims to reduce energy prices, then they should focus on the subsidies for energy investments. On the other hand, if the government considers households' welfare as the most important objective, then they should consider energy vouchers to all households (not only to low-income households).

Table 3: Welfare analysis, % deviation from baseline

Energy policies	Welfare effects
energy vouchers to all households, 1% GDP	-5.07E-6
energy vouchers to low-income households only, 1% GDP	-2.94E-3
subsidies for energy investment, 1% GDP	-1.41E-3
energy vouchers to firms, 1% GDP	-1.77E-2

7 Conclusion

The present paper studies the potential macroeconomic effects of various fiscal policy incentives following a negative energy supply shock. To this end, we build a heterogeneous agent DSGE model where energy production is endogenous. To the best of our knowledge, this is the first paper to study the effects of such a policy within a DSGE framework. Both Ricardian and hand-to-mouth households populate the model, capturing a reality that some households do not save, and thus are more vulnerable to negative economic shocks as they can't hedge against sudden price increases.

In numerical experiments, we study the macroeconomic effects of four fiscal policies that have either been proposed or implemented in the European Union: energy vouchers that are directed to either all households, low-income households, or to non-energy producers, and a subsidy for investments in the capital sector. Since the energy price increase follows from a supply shock, the latter is the most effective to reduce the energy price in the short term. However, this policy is welfare-dominated by vouchers to households.

Any policy that redistributes income to low-income households will reduce total savings in the economy as a fraction of low-income households never save by assumption.

We conclude that if the government only targets lower energy prices, then they should fo-

cus on the subsidies for energy investment. If the government wants to help alleviate the increased living costs and energy poverty, they should instead consider energy vouchers to all households. According to our results, an energy voucher given directly to non-energy firms is inferior to the other policies both when it comes to reducing the energy price, and in terms of welfare.

The findings in this paper raises several suggestions for future research. First, in our model, and as is standard in the literature, investments are marginal. In reality, however, investments in energy production are typically lumpy and non-marginal, and dominated by large producers (increasing capacity by building new nuclear- or hydropower plants are good examples). Such investments take a long time to realize. It is likely that this type of lumpiness in the capacity will reduce the effectiveness of investment subsidies in reducing the energy price in the short- to medium term. Rather, we would expect that big push-type investments are necessary to ensure that capacity is expanded to the degree where it has a real impact on energy prices. Future research concerning investment subsidies to mitigate the effects of supply shocks should take this into account.

Similarly, while our analysis suggests that it is better to subsidize all households than to target low-income households, this ignores the fact that some households with limited liquidity are unable to cope with high energy prices even for a short time period. While this is partially accounted for through the inclusion of hand-to-mouth consumers in our model, we do not explicitly model a minimum subsistence level of energy consumption as we assume standard CRRA preferences. The positive welfare effects of reducing the burden on low-income households are therefore likely downward biased.

References

- Afshan, S., Zaiied, Y. B., Yaqoob, T., and Managi, S. (2024). Insights into the efficiency of china’s green energy policies. *Journal of Cleaner Production*, 434:139913.
- Amjadi, G., Lundgren, T., and Persson, L. (2018). The rebound effect in swedish heavy industry. *Energy Economics*, 71:140–148.
- Amundsen, E. S. and Bergman, L. (2006). Why has the nordic electricity market worked so well? *Utilities policy*, 14(3):148–157.
- Antunes, M., Teotónio, C., Quintal, C., and Martins, R. (2023). Energy affordability across and within 26 european countries: Insights into the prevalence and depth of problems using microeconomic data. *Energy Economics*, 127:107044.
- Ari, M. A., Arregui, M. N., Black, M. S., Celasun, O., Iakova, M. D. M., Mineshima, M. A., Mylonas, V., Parry, I. W., Teodoru, I., and Zhunussova, K. (2022). *Surging energy prices in europe in the aftermath of the war: How to support the vulnerable and speed up the transition away from fossil fuels*. International Monetary Fund.
- Auclert, A., Monnery, H., Rognlie, M., and Straub, L. (2023). Managing an energy shock: Fiscal and monetary policy. Technical report, Working Paper.
- Balke, N. S. and Brown, S. P. (2018). Oil supply shocks and the us economy: An estimated dsge model. *Energy policy*, 116:357–372.
- Bayer, C., Kriwoluzky, A., Müller, G. J., and Seyrich, F. (2023). Hicks in hank: Fiscal responses to an energy shock.
- Blanchard, O. J. and Bernanke, B. S. (2023). What caused the us pandemic-era inflation? Technical report, National Bureau of Economic Research.
- Böhringer, C., García-Muros, X., and González-Eguino, M. (2022). Who bears the burden of greening electricity? *Energy Economics*, 105:105705.
- Broberg, T., Berg, C., and Samakovlis, E. (2015). The economy-wide rebound effect from improved energy efficiency in swedish industries—a general equilibrium analysis. *Energy policy*, 83:26–37.
- Cai, Y., Zhang, D., Chang, T., and Lee, C.-C. (2022). Macroeconomic outcomes of opec and non-opec oil supply shocks in the euro area. *Energy Economics*, 109:105975.
- Cashin, P., Mohaddes, K., Raissi, M., and Raissi, M. (2014). The differential effects of oil demand and supply shocks on the global economy. *Energy Economics*, 44:113–134.
- Chan, J., Diz, S., and Kanngiesser, D. (2022). Energy prices and household heterogeneity: Monetary policy in a gas-tank. *Available at SSRN 4255158*.
- Dahlqvist, A., Lundgren, T., and Marklund, P.-O. (2021). The rebound effect in energy-intensive industries: A factor demand model with asymmetric price response. *The Energy Journal*, 42(3).

- Dhawan, R. and Jeske, K. (2008). Energy price shocks and the macroeconomy: the role of consumer durables. *Journal of Money, Credit and Banking*, 40(7):1357–1377.
- Edquist, H. and Henrekson, M. (2017). Swedish lessons: How important are ict and r&d to economic growth? *Structural Change and Economic Dynamics*, 42:1–12.
- EU-commission (2022a). Press release: Council formally adopts emergency measures to reduce energy prices.
- EU-commission (2022b). Tackling rising energy prices: a toolbox for action and support.
- Friedrich, B., Laun, L., and Meghir, C. (2021). Income dynamics of immigrants and natives in sweden 1985-2016. Technical report, National Bureau of Economic Research.
- Gagliardone, L. and Gertler, M. (2023). Oil prices, monetary policy and inflation surges. Technical report, National Bureau of Economic Research.
- Gonzalez-Torres, M., Bertoldi, P., Castellazzi, L., and Pérez-Lombard, L. (2023). Review of eu product energy efficiency policies: What have we achieved in 40 years? *Journal of Cleaner Production*, page 138442.
- Halkos, G. and Zisiadou, A. (2023). Energy crisis risk mitigation through nuclear power and res as alternative solutions towards self-sufficiency. *Journal of Risk and Financial Management*, 16(1):45.
- Hamilton, J. D. (1983). Oil and the macroeconomy since world war ii. *Journal of political economy*, 91(2):228–248.
- Hamilton, J. D. (1988). A neoclassical model of unemployment and the business cycle. *Journal of political Economy*, 96(3):593–617.
- Hamilton, J. D. (2008). Oil and the macroeconomy. *The new Palgrave dictionary of economics*, 2.
- Kilian, L. (2008). The economic effects of energy price shocks. *Journal of economic literature*, 46(4):871–909.
- Kim, I.-M. and Loungani, P. (1992). The role of energy in real business cycle models. *journal of Monetary Economics*, 29(2):173–189.
- Langot, F., Malmberg, S., Tripier, F., and Hairault, J.-O. (2023). The macroeconomic and redistributive effects of shielding consumers from rising energy prices: the french experiment. Technical report, CEPREMAP.
- Lorenzoni, G. and Werning, I. (2023). Inflation is conflict. Technical report, National Bureau of Economic Research.
- Lucas Jr, R. E. (1990). Supply-side economics: An analytical review. *Oxford economic papers*, 42(2):293–316.
- OECD (2023). Income distribution database.
- Olovsson, C. (2009). Why do europeans work so little? *International Economic Review*,

50(1):39–61.

- Papież, M., Śmiech, S., and Frodyma, K. (2021). The role of energy policy on the decoupling processes in the european union countries. *Journal of Cleaner Production*, 318:128484.
- Pieroni, V. (2023). Energy shortages and aggregate demand: Output loss and unequal burden from hank. *European Economic Review*, 154:104428.
- Plante, M. (2014). The long-run macroeconomic impacts of fuel subsidies. *Journal of Development Economics*, 107:129–143.
- Schmalensee, R. (2012). Evaluating policies to increase electricity generation from renewable energy. *Review of Environmental Economics and Policy*.
- Trujillo-Baute, E., del Río, P., and Mir-Artigues, P. (2018). Analysing the impact of renewable energy regulation on retail electricity prices. *Energy Policy*, 114:153–164.
- UN (2022). World economic situation and prospects: December 2022 briefing, no. 167.
- van de Ven, D. J. and Fouquet, R. (2017). Historical energy price shocks and their changing effects on the economy. *Energy Economics*, 62:204–216.
- von Platten, J. (2022a). Energifattigdom i sverige. In *Du sköra nya värld*, pages 195–214. SOM-institutet, Göteborgs universitet.
- von Platten, J. (2022b). Energy poverty in sweden: Using flexibility capital to describe household vulnerability to rising energy prices. *Energy Research & Social Science*, 91:102746.
- Wang, H., Guo, Y., Guo, H., and Safar, B. (2021). Energy policy influence on the surplus welfare of subscribers in present of cleaner energy sources. *Journal of Cleaner Production*, 280:124145.
- Wildauer, R., Kohler, K., Aboobaker, A., and Guschanski, A. (2023). Energy price shocks, conflict inflation, and income distribution in a three-sector model. *Energy Economics*, 127:106982.
- Yau, R. and Chen, G.-H. (2021). Assessing energy subsidy policies in a structural macroeconomic model. *Energy Economics*, 103:105509.

A IRFs of BAU

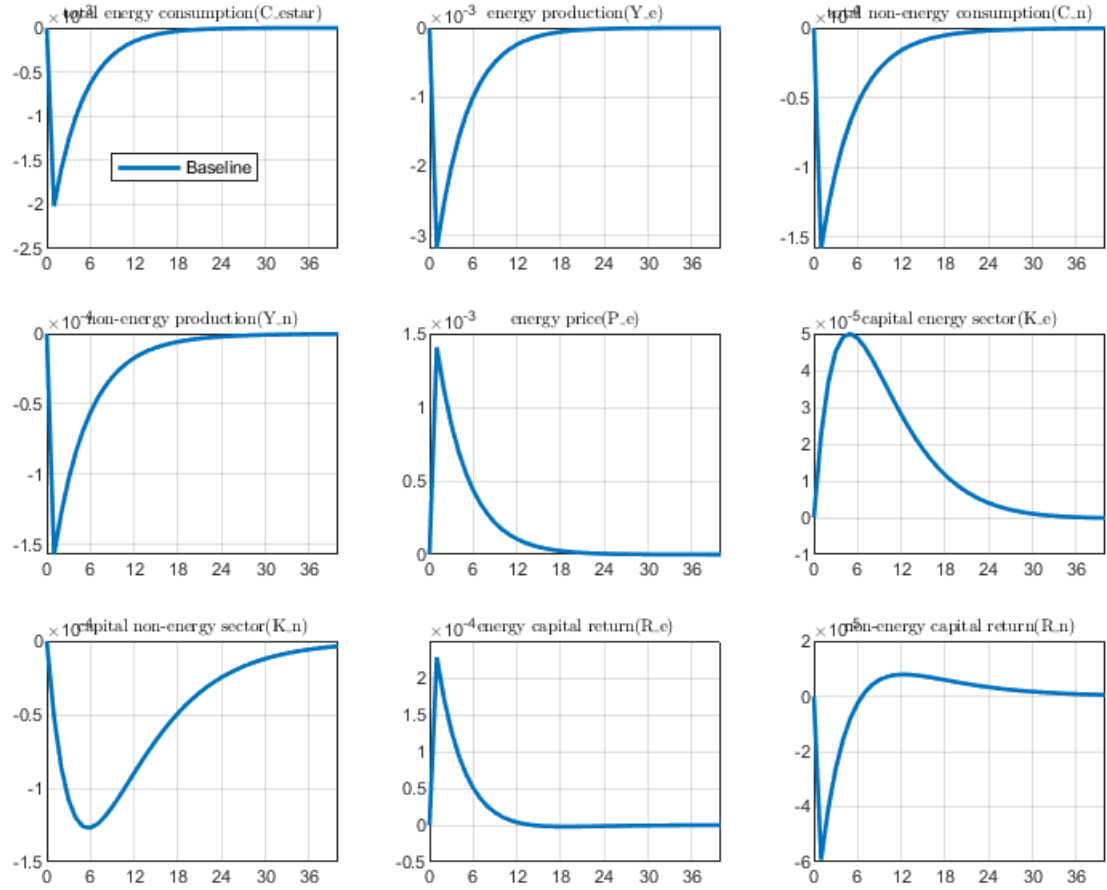


Figure 6: Business-As-Usual scenario, with -1pp shock on the productivity in energy sector

- 1

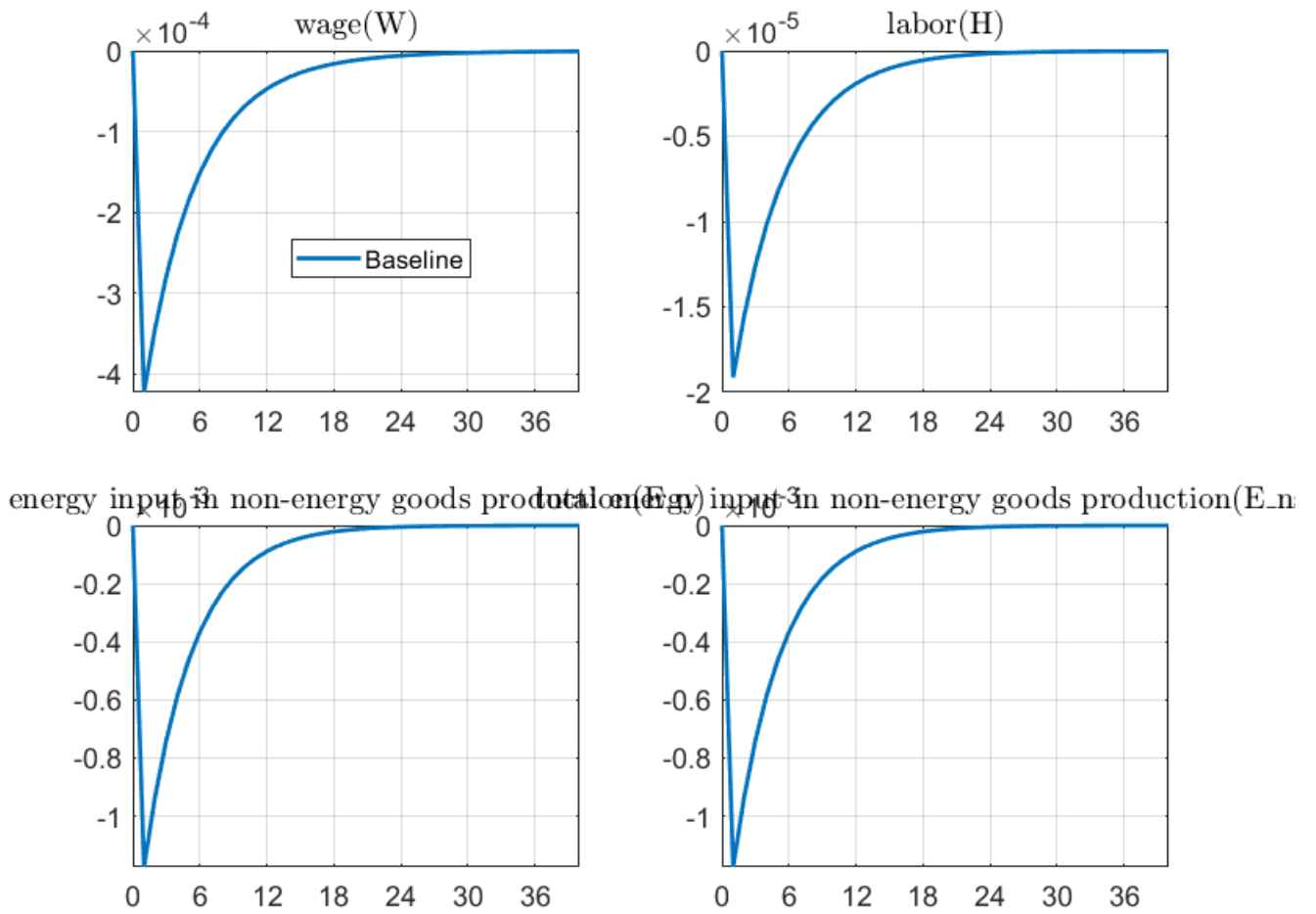


Figure 7: Business-As-Usual scenario, with -1pp shock on the productivity in energy sector
- 2

B Net effects of 4 fiscal instruments

An alternative way to illustrate the effects of the different policies is to compare them relative to BAU, rather than comparing them to steady state. As before, the fiscal budget is 1% of GDP. Figures 8 and 9 show the results.

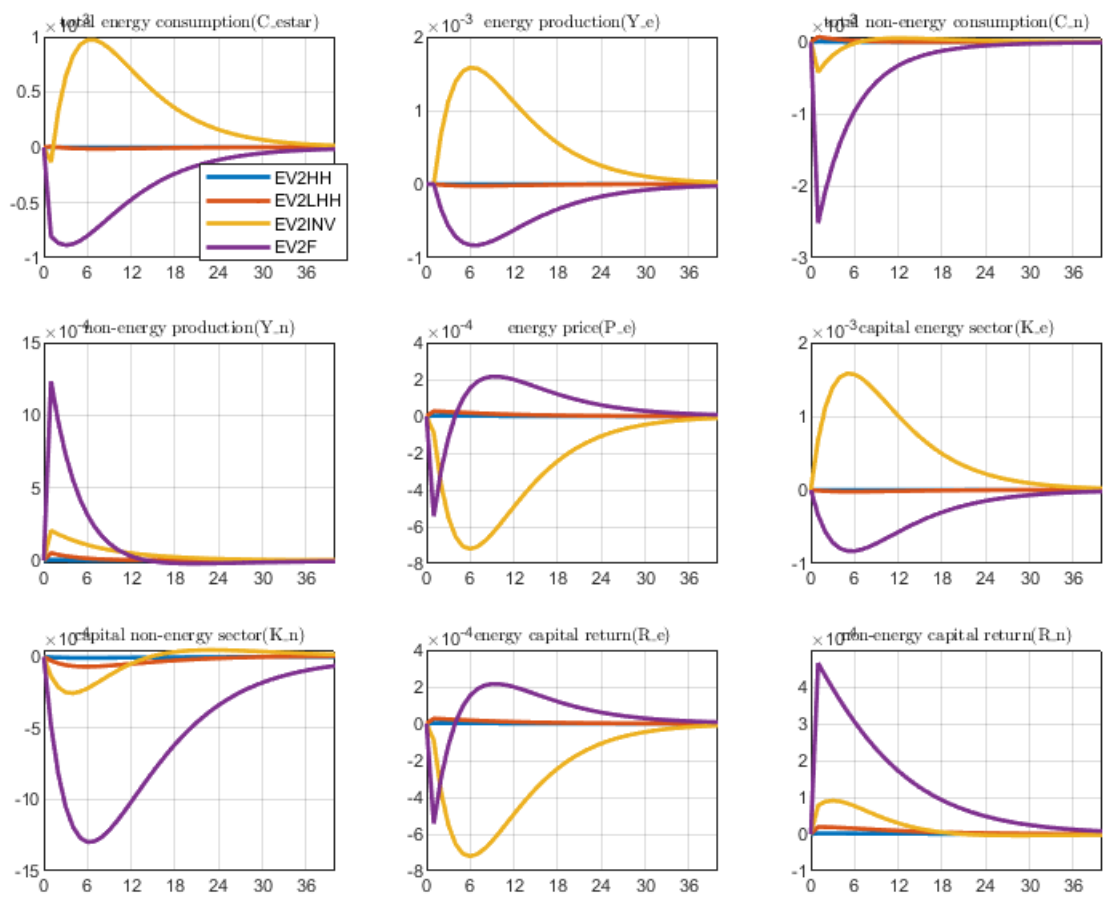


Figure 8: Net effects of 4 fiscal instruments, with the same budget as 1% of GDP - 1

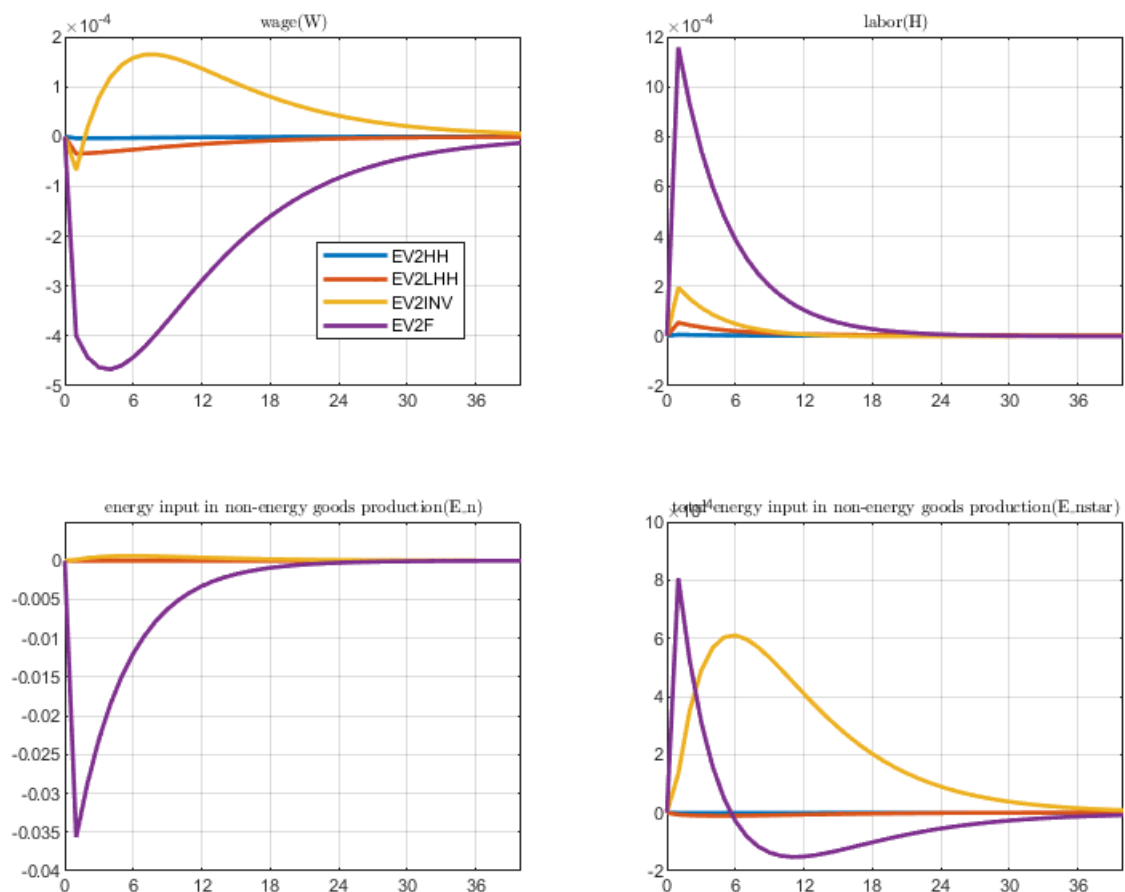


Figure 9: Net effects of 4 fiscal instruments, with the same budget as 1% of GDP - 2

C Sensitivity test on the capital adjustment cost

In this section, we study the sensitivity of our results to changes in the capital adjustment cost parameters. The results are presented in figures 10 to 12. First, figure 10 shows the simulation with smaller capital adjustment cost parameters: $\phi_e = \phi_n = 5$.

Evidently, we see that the main results do not change; i.e., the subsidy for investment in the energy sector is the most effective to bring the energy price down and increase the production. Next, figure 11 shows the simulation with larger capital adjustment cost parameters equal to $\phi_e = \phi_n = 20$, and figure 12 shows the simulation with even larger capital adjustment cost parameters of $\phi_e = \phi_n = 50$. We see that as we increase the values of ϕ_e and ϕ_n , i.e., when the capital adjustment becomes more costly, the effects from fiscal policies become less evident. Concerning the energy price $P_{e,t}$, when the capital adjustment becomes more costly, it takes more time for the energy supply effect to dominate and there is more and more

delay to the decline of the energy price compared to BAU. In the extreme case, where we set $\phi_e = \phi_n = 500$, the investment subsidy (the purple curve) has almost no effect compared to BAU. As for the scenario with extremely large capital adjustment cost, the effects from equation 9 become dominant and the energy price is lower than in the BAU. Nevertheless, it is not effective to increase households energy consumption nor energy production.

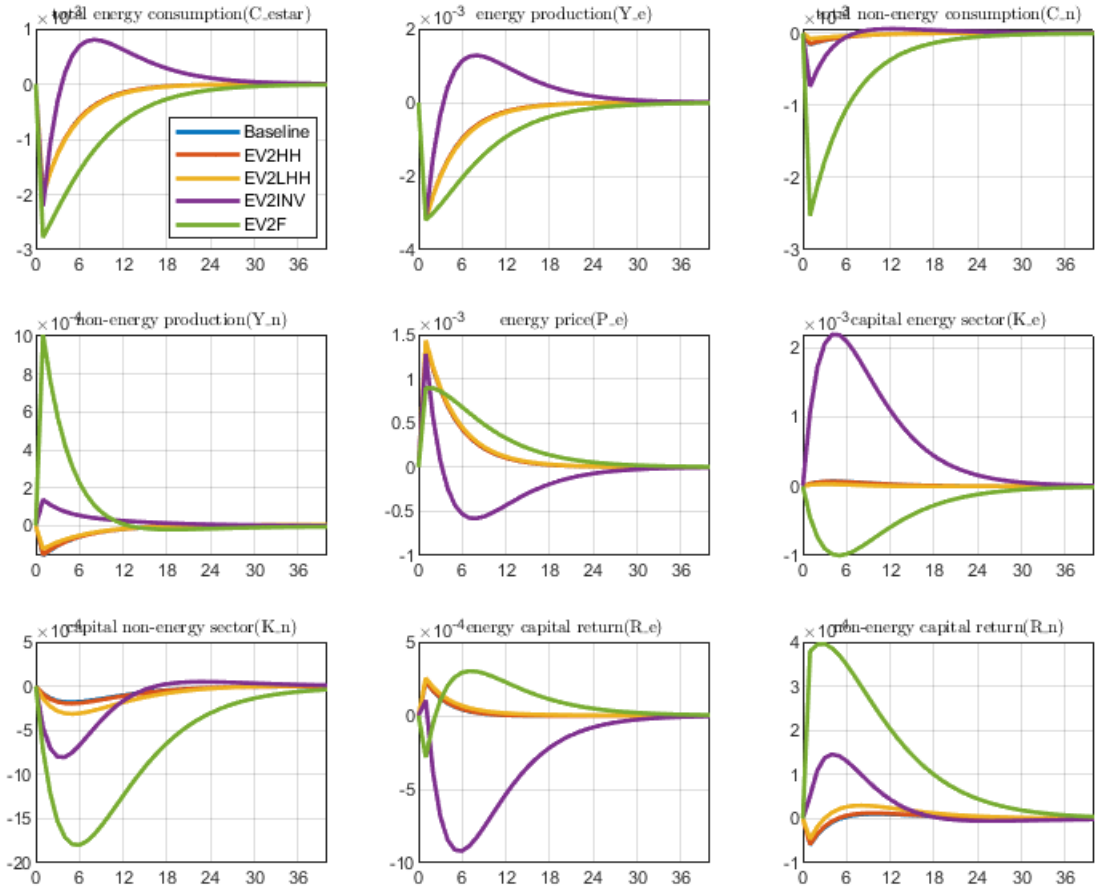


Figure 10: Effects of 4 fiscal instruments and BAU, with the same budget as 1% of GDP, $\phi_e = \phi_n = 5$

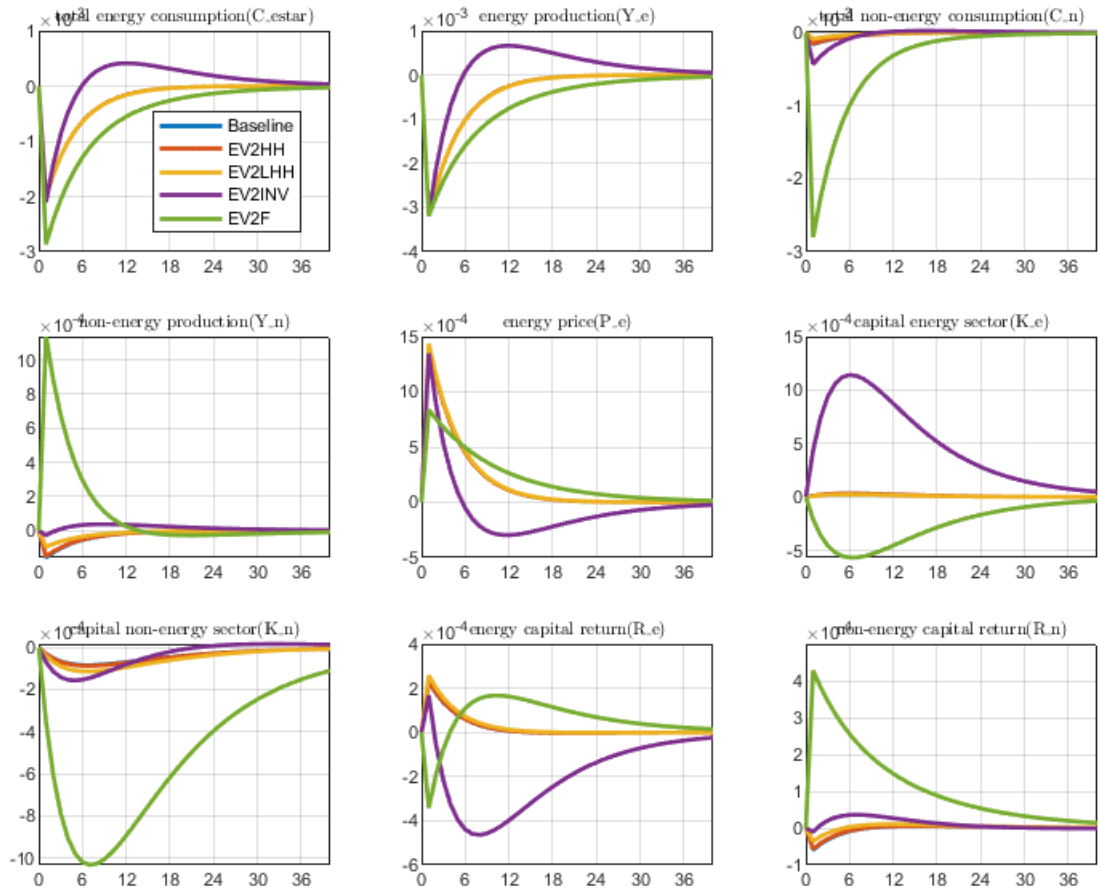


Figure 11: Effects of 4 fiscal instruments and BAU, with the same budget as 1% of GDP, $\phi_e = \phi_n = 20$

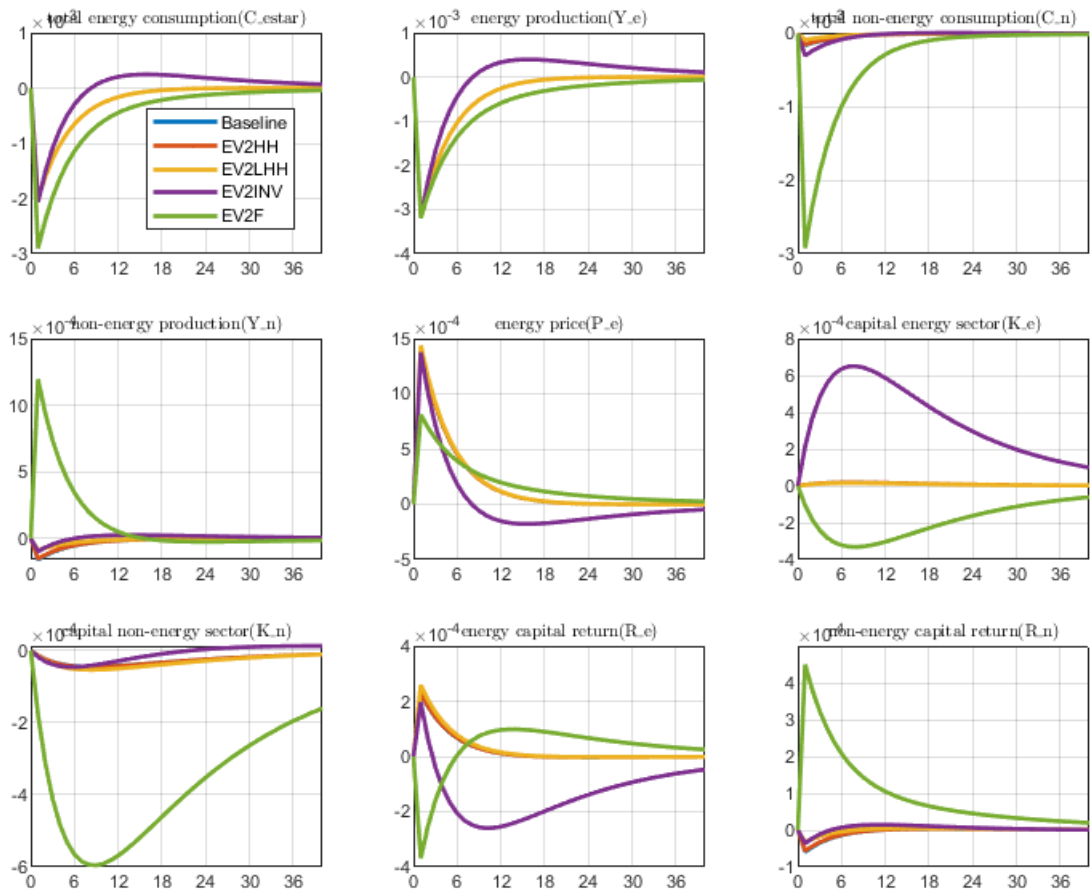


Figure 12: Effects of 4 fiscal instruments and BAU, with the same budget as 1% of GDP, $\phi_e = \phi_n = 50$

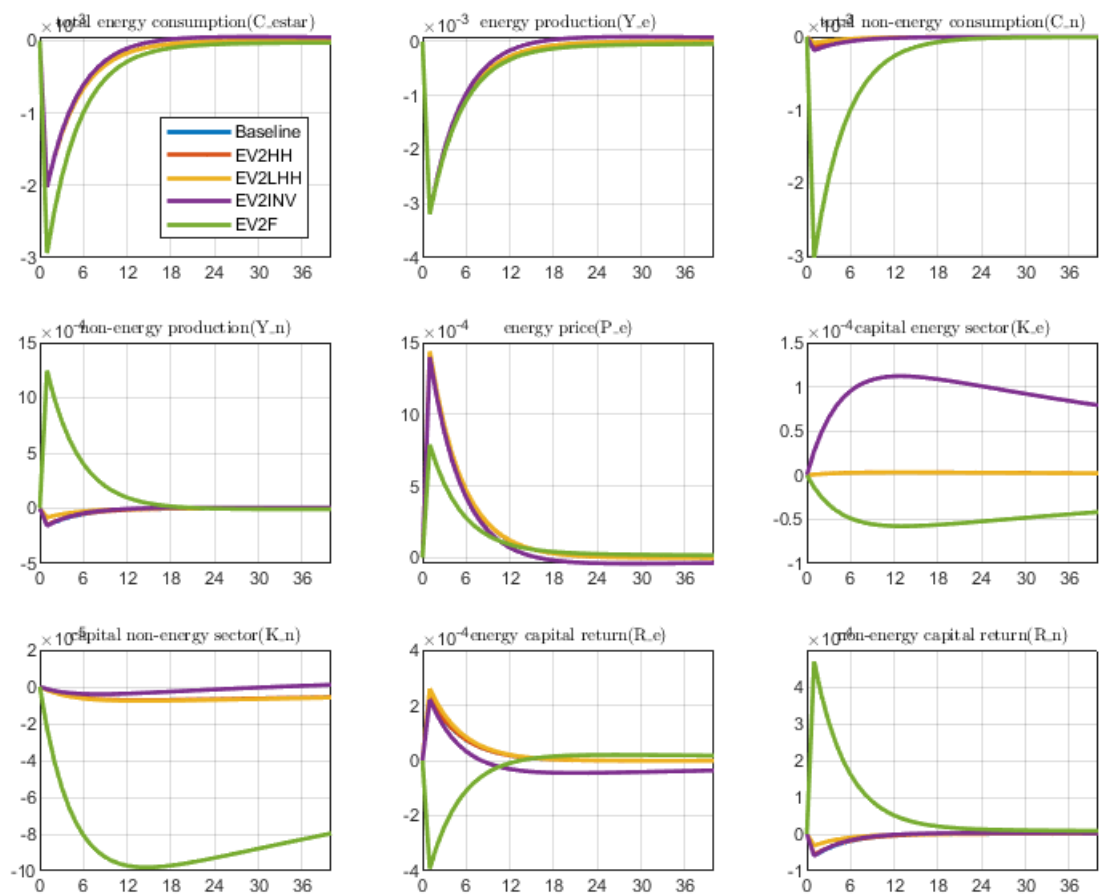


Figure 13: Effects of 4 fiscal instruments and BAU, with the same budget as 1% of GDP, $\phi_e = \phi_n = 500$

Last, we perform the welfare analysis under the different values assumed for the capital adjustment costs. These results are presented in Table 4. In all the scenarios, the welfare effects show that the energy voucher to firms generate the largest welfare loss, and the energy voucher to all household generates the least welfare loss. The welfare loss from the subsidy for energy investment decreases with capital adjustment cost. In other words, when there is more friction in the capital market, the investment subsidy welfare loss of the subsidy for capital investments in the energy sector becomes smaller.

Table 4: Welfare analysis, % deviation from baseline

Energy policies, values of ϕ_e, ϕ_n	5	10	20	50	500
energy vouchers to all households, 1% GDP	-1.64E-7	-5.55E-6	-9.45E-6	-1.25E-5	-1.51E-5
energy vouchers to low-income households only, 1% GDP	-1.42E-3	-1.61E-3	-1.75E-3	-1.86E-3	-1.96E-3
subsidies for energy investment, 1% GDP	-2.37E-3	-1.67E-3	-1.11E-3	-5.95E-4	-1.20E-4
energy vouchers to firms, 1% GDP	-1.18E-2	-1.22E-2	-1.25E-2	-1.29E-2	-1.33E-2

D Calculation of resource constraint

We add the budget constraints of households weighted by the proportion of each household, we get

$$\begin{aligned}
& C_{n,t} + P_{e,t}C_{e,t} + INV_{n,t} + (1 - s_{e,t})INV_{e,t} + \sum_{i=1,2} \Lambda_i^{RS} \frac{\psi_n}{2} (k_{n,i,t}^{RS} - k_{n,i,t-1}^{RS})^2 \\
& + \sum_{i=1,2} \Lambda_i^{RS} \frac{\psi_e}{2} (k_{e,i,t}^{RS} - k_{e,i,t-1}^{RS})^2 = (1 - \omega)W_tH_t + T_t + R_{n,t}K_{n,t-1} + R_{e,t}K_{e,t-1} \quad (40)
\end{aligned}$$

In the RHS of the equation above, we substitute the government transfer T_t by the budget constrain of the government, we get :

$$\begin{aligned}
& C_{n,t} + P_{e,t}C_{e,t} + INV_{n,t} + (1 - s_{e,t})INV_{e,t} + \sum_{i=1,2} \Lambda_i^{RS} \frac{\psi_n}{2} (k_{n,i,t}^{RS} - k_{n,i,t-1}^{RS})^2 \\
& + \sum_{i=1,2} \Lambda_i^{RS} \frac{\psi_e}{2} (k_{e,i,t}^{RS} - k_{e,i,t-1}^{RS})^2 \\
& = \omega W_t H_t - (T_{h,e,t} + T_{n,e,t} + s_{e,t}INV_{e,t}) + (1 - \omega)W_t H_t + R_{n,t}K_{n,t-1} + R_{e,t}K_{e,t-1} \\
& = W_t H_t - (T_{h,e,t} + T_{n,e,t} + s_{e,t}INV_{e,t}) + R_{n,t}K_{n,t-1} + R_{e,t}K_{e,t-1}
\end{aligned}$$

We then substitute the labor and capital costs on the RHS by the zero profit conditions of energy and non-energy firms :

$$\begin{aligned}
RHS & = Y_{n,t} - R_{n,t}K_{n,t-1} - P_{e,t}E_{n,t} + R_{n,t}K_{n,t-1} + P_{e,t}Y_{e,t} - (T_{h,e,t} + T_{n,e,t} + s_{e,t}INV_{e,t}) \\
& = Y_{n,t} + P_{e,t}Y_{e,t} - P_{e,t}E_{n,t} - (T_{h,e,t} + T_{n,e,t} + s_{e,t}INV_{e,t})
\end{aligned}$$

Finally we have the LHS = RHS in equation 40, i.e.

$$\begin{aligned}
& C_{n,t} + P_{e,t}C_{e,t} + INV_{n,t} + (1 - s_{e,t})INV_{e,t} + \sum_{i=1,2} \Lambda_i^{RS} \frac{\psi_n}{2} (k_{n,i,t}^{RS} - k_{n,i,t-1}^{RS})^2 \\
& + \sum_{i=1,2} \Lambda_i^{RS} \frac{\psi_e}{2} (k_{e,i,t}^{RS} - k_{e,i,t-1}^{RS})^2 \\
& = Y_{n,t} + P_{e,t}Y_{e,t} - P_{e,t}E_{n,t} - (T_{h,e,t} + T_{n,e,t} + s_{e,t}INV_{e,t})
\end{aligned}$$

which is equivalent to (the final resource constraint)

$$\begin{aligned}
& Y_{n,t} + P_{e,t}Y_{e,t} = C_{n,t} + P_{e,t}C_{e,t} + P_{e,t}E_{n,t} + INV_{n,t} + INV_{e,t} + T_{h,e,t} + T_{n,e,t} \\
& + \sum_{i=1,2} \Lambda_i^{RS} \frac{\psi_n}{2} (k_{n,i,t}^{RS} - k_{n,i,t-1}^{RS})^2 + \sum_{i=1,2} \Lambda_i^{RS} \frac{\psi_e}{2} (k_{e,i,t}^{RS} - k_{e,i,t-1}^{RS})^2 \quad (41)
\end{aligned}$$