STAFF MEMO

Financial factors and the macroeconomy
- a policy model

NORGES BANK

Financial Factors and the Macroeconomy - A Policy Model

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Abstract

This paper documents the theoretical structure of an extension of the Norges Bank policy model NEMO. New features include an explicit treatment of the credit market, including a separate banking sector, a role for housing services and house prices, and the option of using macro-prudential instruments as the LTV-ratio and capital requirements as policy instruments. The model rely on building blocks from the recent literature on the interaction between the financial sector and the real economy.

Keywords: DSGE, credit market frictions, banking sector.

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1 Introduction

This paper documents the theoretical structure of an extension of the Norges Bank policy model NEMO.[1] New features include an explicit treatment of the credit market, including a separate banking sector, a role for housing services and house prices, and the option of using macro-prudential instruments as the LTV-ratio and capital requirements as policy instruments. The model rely on building blocks from the recent literature on the interaction between the financial sector and the real economy.[2]

The model was introduced for policy analysis in 2013, and is a preliminary output from an ongoing project at the Norges Bank aimed at deepening our understanding of the linkages between financial factors and the rest of the economy, including monetary policy. This is important for several reasons. First, a more appropriate modelling of the credit channel is of first order importance to monetary policy in its own right. To the extent that there are significant feed-backs from the financial sector to the real economy, which the great recession seems to indicate, this should be internalized by monetary policy. Moreover, reducing misspecification in the model will improve the structural interpretation of the data. Second, a more explicit modelling of financial variables relevant for financial stability will make it easier to discuss policy implications of potential financial imbalances building up. Third, including macro-prudential instruments in the model, also allows us to analyze issues related to the interaction between monetary policy and macroprudential policies.

Building and improving models for policy analysis is a continuos process. Work is already underway to improve several aspects of the model. First, we will allow for mulitiperiod debt contracts.[3] This will give rise to more persistent financial cycles, which is a prominent feature of the data. Second, we plan to relax the current assumption that banks can only finance themselves domestically through deposits. On the margin,

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[1] See Brubakk et al. (2006) and Brubakk and Sveen (2009).

[2] In particular, we benefit from contributions by Iacoviello (2005), Gerali et al. (2010) and Benes and Kumhof (2011).

[3] See Kydland et al. (2012), Garriga et al. (2013), Gelain et al. (2014a), and Gelain et al. (2014b).

international funding seems to be an important channel of bank funding. Third, we also plan to relax the assumption of rational expectations by introducing so-called hybrid expectations,[4] which yields more intrinsic persistence in line with VAR evidence. Finally, an important future extension will be to explicitly model the apparent non-linear relation between financial market developments and episodes of financial stress.

All future changes to the model will be documented in updated versions of this StaffMemo. Once the aforementioned improvements are implemented and the model reestimated, a detailed companion paper showing impulse responses and various policy analysis will be published.

2 The model

2.1 Main features

The model economy consists of households, firms and a government sector, including the monetary authority. There are two main production sectors, an *intermediate goods sector* and a *final goods sector*. In addition, there are two separate production sectors for housing and non-housing capital goods, partly relaying on final goods as inputs.

Each intermediate good is produced by a single firm, using differentiated labor, l, and capital, K, as inputs. The market for intermediate goods is characterized by monopolistic competition. The intermediate good, T, can be exported or sold domestically to the final goods sector. Under the assumption of monopolistic competition, intermediate firms will set their prices as a mark-up over marginal costs. Since we abstract from the possibility of arbitrage across countries, intermediate firms can set different prices at home and abroad. Furthermore, we assume that it is costly for intermediate firms to change their prices. The specification of the price adjustment costs is consistent with Rotemberg (1982). Intermediate firms rent capital from a separate set of firms called entrepreneurs. Entrepreneurs rely on external funding from the banking sector

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[4]Crudely speaking, *hybrid expectations* refers to a weighted average of backward-looking moving average (MA) terms and a forward-looking component (See Gelain et al. (2013)).

to finance capital investments.

In the final goods sector, domestic and imported intermediate goods, Q and M respectively, are combined to produce a final retail good, A. Firms in this sector are assumed to operate under perfect competition. The final good can be used for consumption, C, capital production, I, housing production, I^H, government spending, G, and oil investment, I^{oil}.[5]

We assume that there are two types of households in the economy, patient and impatient. Both types choose the level of housing services, non-housing consumption, wages and credit in order to maximize their utility given an intertemporal budget restriction and the demand schedule for labor. Borrowing from the banking sector is constrained by the value of the housing stock, which serves as collateral. Since by assumption, impatient households have a higher time preference rate than patient households, the borrowing constraint will only apply to impatient households. Hence, in equilibrium, impatient households borrow, while patent households save.

The banking sector is modelled mainly on the basis of two reference papers, namely Gerali et al. (2010) and Benes and Kumhof (2011). The structure of the banking sector is from Gerali et al. (2010). It has three distinctive features. First, banks enjoy some degree of market power in loan and deposit markets and set different rates for households and firms. Second, banks face costs of adjusting retail rates and the pass-through on loan and deposit rates of changes in the policy rate is incomplete on impact. Third, differently from Gerali et al. (2010), we assume that bank capital is subject to regulation as in Benes and Kumhof (2011). They see regulation as a system of penalties imposed on banks in case they fall below the regulatory minimum.

Government spending is financed through lump-sum tax revenues. The monetary authority controls the national short-term nominal interest rate. The optimal interest rate path is obtained by minimizing a loss function, where the inflation and output gap are the main arguments. Monetary policy ensures that the steady state inflation rate

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[5]We model the mainland economy, that is, the total economy excluding the oil sector. However, whereas oil production is not modeled, we include (exogenously) oil investments on the demand side, affecting mainland industries.

is equal to the inflation target.

We assume that the economy evolves along a balanced growth path, driven by two exogenous productivity shocks. We adopt the small open economy assumption, implying that the foreign economy (rest of the world) is fully exogenous from the point of view of the Norwegian economy. Hence, economic developments in Norway have no effects on its trading partners. This is a reasonable description, given the relative size of the Norwegian economy.[6]

2.2 Final goods

We assume that there is a continuum of final good producers indexed by $x \in [0, 1]$. The final good, A, is produced using a composite of domestic intermediate goods, Q, and imports, M, as inputs. The specific technology adopted is a constant elasticity of substitution (CES) production function:

$$A_t(x) = \left[\eta^{\frac{1}{\mu}} Q_t(x)^{1-\frac{1}{\mu}} + (1-\eta)^{\frac{1}{\mu}} M_t(x)^{1-\frac{1}{\mu}} \right]^{\frac{\mu}{\mu-1}}, \tag{1}$$

where the degree of substitutability between the indices of domestic and imported goods is determined by the parameter $\mu > 0$, whereas η $(0 \leq \eta \leq 1)$ measures the steady-state share of domestic intermediates in the case where relative prices are equal to 1. Hence, the latter is often interpreted as the degree of home bias.

Furthermore, the composite good Q is an index of differentiated domestic intermediate goods, produced by a continuum of firms $h \in [0, 1]$:

$$Q_t(x) = \left[\int_0^1 Q_t(h, x)^{1-\frac{1}{\theta_t^H}} \, dh \right]^{\frac{\theta_t^H}{\theta_t^H - 1}}, \tag{2}$$

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[6]Sutherland (2005) shows how to collapse a fully endogenous two-country model into a model where one of the countries adhere to the small open economy assupmtion, whereas the other country (the "rest of the world") behaves like a closed economy. See also Brubakk et al. (2006). The small open economy assumption is obtained as a limiting case where the relative size of one of the countries goes to zero.

where the degree of substitution between domestic intermediates is captured by $\theta^H > 1$. We allow this parameter to be time varying according to:

$$\ln\left(\frac{\theta^H_t}{\theta^H}\right) = \lambda^{\theta^H} \ln\left(\frac{\theta^H_{t-1}}{\theta^H}\right) + \varepsilon^{\theta^H}_t \qquad 0 \leq \lambda^{\theta^H} < 1, \quad \varepsilon^{\theta^H}_t \sim iid\left(0, \sigma^2_{\theta^H}\right), \quad (3)$$

where θ^H is the steady state value and λ^{θ^H} is the autocorrelation coefficient, determining the persistence of the shock process. The error term $\varepsilon^{\theta^H}_t$ is assumed to be white noise.

Similarly, the composite imported input is an aggregate of differentiated import goods indexed $f \in [0, 1]$:

$$M_t(x) = \left[\int_0^1 M_t\left(f, x\right)^{1-\frac{1}{\theta^F_t}} df\right]^{\frac{\theta^F_t}{\theta^F_t - 1}}, \qquad (4)$$

where $\theta^F > 1$ is the degree of substitution between imported goods. The elasticity of substitution across differentiated imports evolves according to:

$$\ln\left(\frac{\theta^F_t}{\theta^F}\right) = \lambda^{\theta^F} \ln\left(\frac{\theta^F_{t-1}}{\theta^F}\right) + \varepsilon^{\theta^F}_t \qquad 0 \leq \lambda^{\theta^F} < 1, \quad \varepsilon^{\theta^F}_t \sim iid\left(0, \sigma^2_{\theta^F}\right), \quad (5)$$

The demand for the different varieties of domestic goods, $Q\left(h, x\right)$, is obtained by minimizing total outlays on domestic intermediate goods given (2). This yields the following demand functions:

$$Q_t\left(h, x\right) = \left(\frac{P^Q_t\left(h\right)}{P^Q_t}\right)^{-\theta^H_t} Q_t\left(x\right), \qquad (6)$$

where $P^Q_t\left(h\right)$ denotes the price of variety h produced at home and P^Q_t is the corre-

sponding aggregate price,[7] given by:

$$P_t^Q = \left[\int_0^1 P_t^Q (h)^{1-\theta_t^H} \, dh \right]^{\frac{1}{1-\theta_t^H}}. \tag{7}$$

In a similar fashion, the demand for differentiated imports is given by:

$$M_t(x, f) = \left(\frac{P_t^M (f)}{P_t^M} \right)^{-\theta_t^F} M_t (x), \tag{8}$$

where $P_t^M (f)$ denotes the price of imported variety f and P_t^M is the aggregate import price:

$$P_t^M = \left[\int_0^1 P_t^M (f)^{1-\theta_t^F} \, df \right]^{\frac{1}{1-\theta_t^F}}. \tag{9}$$

The optimal choice of $Q_t(x)$ and $M_t(x)$ can be found by minimizing $P_t^Q Q_t(x) + P_t^M M_t(x)$ given (1). This yields the following demand functions:

$$Q_t(x) = \eta \left(\frac{P_t^Q}{P_t} \right)^{-\mu} A_t(x), \tag{10}$$

$$M_t(x) = (1 - \eta) \left(\frac{P_t^M}{P_t} \right)^{-\mu} A_t(x), \tag{11}$$

where P_t is the aggregate price of the final good. The final goods sector is characterized by perfect competition, implying that profits are zero:

$$P_t A_t(x) = P_t^Q Q_t (x) + P_t^M M_t (x). \tag{12}$$

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[7] Defined as the minimal outlay required to produce one unit of the composite.

2.3 Intermediate goods

Each intermediate firm n is assumed to produce a differentiated good $T_t(n)$ for the domestic and the foreign market using the following CES production function:

$$T_t(n) = \left[(1-\alpha)^{\frac{1}{\xi}} \left(Z_t z_t^L l_t(n) \right)^{1-\frac{1}{\xi}} + \alpha^{\frac{1}{\xi}} \overline{K}_t(n)^{1-\frac{1}{\xi}} \right]^{\frac{\xi}{\xi-1}}, \tag{13}$$

where $\alpha \in [0,1]$ is the capital share and ξ denotes the elasticity of substitution between labor and capital. The variables $l_t(n)$ and $K_t(n)$ denote, respectively, hours used and capital of firm n in period t. There are two exogenous shocks to productivity in the model: Z_t refers to an exogenous permanent (level) technology process, which grows at the gross rate π_t^z, whereas z_t^L denotes a temporary (stationary) shock to productivity (or labor utilization). We assume the following processes:

$$\ln(Z_t) = \ln(Z_{t-1}) + \ln(\pi^z) + \ln \frac{\pi_t^z}{\pi^z}, \tag{14}$$

where

$$\ln \frac{\pi_t^z}{\pi^z} = \lambda^z \ln \frac{\pi_{t-1}^z}{\pi^z} + \varepsilon_t^z \qquad 0 \leq \lambda^z < 1, \quad \varepsilon_t^z \sim iid\left(0, \sigma_z^2\right), \tag{15}$$

and

$$\ln \left(\frac{z_t^L}{z^L} \right) = \ln \left(\frac{z_{t-1}^L}{z^L} \right) + \varepsilon_t^{zl} \qquad \varepsilon_t^{zl} \sim iid\left(0, \sigma_{zl}^2\right), \tag{16}$$

Firms choose labor and capital to minimize factor outlays, taking the wages and the rental rate as given. Capital is hired from entrepreneurs at the rental rate R_t^K. The labor input is a Cobb-Douglas aggregate of hours supplied by the two types of households, l^{im} and l^{pa}, respectively:

$$l_t = \left(l_t^{im} \right)^{\varrho} \left(l_t^{pa} \right)^{1-\varrho}$$

where:

$$l_t^{im}(n) = \left[\frac{1}{n} \int_0^\varrho l_t^{im}(n,j)^{1-\frac{1}{\psi_t^{im}}} dj \right]^{\frac{\psi_t^{im}}{\psi_t^{im}-1}}, \tag{17}$$

and:

$$l_t^{pa}(n) = \left[\frac{1}{n} \int_\varrho^1 l_t^{pa}(n,j)^{1-\frac{1}{\psi_t^{pa}}} dj \right]^{\frac{\psi_t^{pa}}{\psi_t^{pa}-1}}, \tag{18}$$

where ψ_t^k denotes the elasticity of substitution between differentiated labour within labor type $k = pa, im$, and evolves according to:

$$\ln\left(\frac{\psi_t^k}{\psi^k}\right) = \lambda^\psi \ln\left(\frac{\psi_{t-1}^k}{\psi^k}\right) + \varepsilon_t^{\psi^k} \qquad 0 \le \lambda^\psi < 1, \quad \varepsilon_t^{\psi^k} \sim iid\left(0, \sigma_{\psi^k}^2\right), \tag{19}$$

Minimizing expenditure on the the two types of labor, taking (17) and (18) into account, gives rise to the following demand functions:

$$l_t^k(n,j) = \left(\frac{W_t^k(j)}{W_t^k}\right)^{-\psi_t^k} l_t^k(n), \quad k = im, pa \tag{20}$$

where $W_t^k(j)$ is the nominal wage chosen by household j, type k, and W_t^k is the aggregate nominal wage for households of type k, defined as the unit cost of the labor input, $l_t^k(n)$.

The optimal use of the two labor aggregates, l_t^{im} and l_t^{pa} is found by minimizing $W_t^{im} l_t^{im} + W_t^{pa} l_t^{pa}$ given (20). This yields an expression for the overall wage level:

$$W_t = \left[\varrho^\varrho (1-\varrho)^{(1-\varrho)}\right] \left(W_t^{im}\right)^\varrho \left(W_t^{pa}\right)^{(1-\varrho)}$$

Minimizing total factor outlays for a given level of production, results in the following first order conditions (in symmetric equilibrium):

$$K_t \equiv \alpha \left(\frac{MC_t}{R_t^K}\right)^\xi T_t. \tag{21}$$

9

and:
$$l_t = (1 - \alpha) \left(\frac{MC_t}{W_t} \right)^{\xi} T_t \left(Z_t z_t^L \right)^{(\xi - 1)}. \tag{22}$$

Using (13), (21) and (22), we can solve for the marginal costs, MC_t, to obtain:

$$MC_t = \left[(1 - \alpha) \left(\frac{W_t}{Z_t z_t^L} \right)^{1 - \xi} + \alpha \left(R_t^K \right)^{1 - \xi} \right]^{\frac{1}{1 - \xi}}$$

Firms sell their goods under monopolistic competition. Each firm n charges different prices at home and abroad: $P_t^Q(n)$ in the home market and $P_t^{M*}(n)$ abroad, where the latter is denoted in foreign currency.[8] Again, we assume that changing prices is costly. When a firm changes its prices it incurs intangible costs that do not affect cash-flow but enter the maximization problem as a form of "disutility". The intangible costs of adjusting prices in the domestic and the foreign market are, respectively:

$$\gamma_t^{PQ}(n) \equiv \frac{\phi^{PQ}}{2} \left[\frac{P_t^Q(n) / P_{t-1}^Q(n)}{P_{t-1}^Q / P_{t-2}^Q} - 1 \right]^2, \tag{23}$$

$$\gamma_t^{PM*}(n) \equiv \frac{\phi^{M*}}{2} \left[\frac{P_t^{M*}(n) / P_{t-1}^{M*}(n)}{P_{t-1}^{M*} / P_{t-2}^{M*}} - 1 \right]^2, \tag{24}$$

where the cost of changing prices is governed by the parameters ϕ^{PQ} and ϕ^{M*}.

Cash-flows in a given period are immediately paid out to shareholders (savers) as dividends, $\Psi_t(h)$:

$$\begin{aligned} \Psi_t(n) &= P_t^Q(n) \int_0^1 Q_t(n, x) dx + P_t^{M*}(n) S_t \int_0^1 M_t^*(n, x^*) dx^* \\ &\quad - W_t l_t(n) - R_t^K(n) \overline{K}_{t-1}(n), \end{aligned} \tag{25}$$

where S_t is the nominal exchange rate.

Given optimal factor inputs, and thus the minimal marginal costs, firms choose

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[8]Hence, we assume "local currency pricing" explored by Devereux and Engel (2003), Corsetti and Dedola (2003) and others.

prices to maximize present discounted value of cash-flows, adjusted for the intangible cost of changing prices, taking into account the demand both at home and abroad, $T_t^D(n)$. The latter is given by:

$$
\begin{aligned}
T_t^D(n) &= \int_0^1 Q_t(n,x)dx + \int_0^1 M_t^*(n,x^*)dx^* \\
&= \left(\frac{P_t^Q(n)}{P_t^Q}\right)^{-\theta_t^H} Q_t(x) + \left(\frac{P_t^{M^*}(n)}{P_t^{M^*}}\right)^{-\theta_t^{F^*}} M_t^*(x).
\end{aligned}
\tag{26}
$$

The first order condition for optimal price setting in the domestic market can be written (symmetric equilibrium):

$$
Q_t - \theta_t^H Q_t + \theta_t^H Q_t MC_t(h)/P_t^Q
$$
$$
-\phi^{PQ}\left[\frac{P_t^Q/P_{t-1}^Q}{P_{t-1}^Q/P_{t-2}^Q} - 1\right] P_t^Q Q_t \frac{1/P_{t-1}^Q}{P_{t-1}^Q/P_{t-2}^Q}
$$
$$
+E_t D_{t,t+1}^{pa}\left\{
\begin{array}{c}
\phi^{PQ}\left[\frac{P_{t+1}^Q(h)/P_t^Q(h)}{P_t^Q/P_{t-1}^Q} - 1\right] \times \\
P_{t+1}^Q Q_{t+1}\left(\frac{1}{P_t^Q}\right)^2 \frac{P_{t+1}^Q}{P_t^Q/P_{t-1}^Q}
\end{array}
\right\} = 0,
\tag{27}
$$

where $D_{t,t+1}^{pa}$ denotes the stochastic discount factor of patient households, defined in (38).

The first-order condition for foreign price setting is given by (again skipping the firm index):

$$
S_t M_t^* - \theta_t^{F^*} S_t M_t^* + \theta_t^{F^*} MC_t M_t^*/P_t^{M^*}
$$
$$
-\phi^{M^*}\left[\frac{P_t^{M^*}/P_{t-1}^{M^*}}{P_{t-1}^{M^*}/P_{t-2}^{M^*}} - 1\right] P_t^{M^*} S_t M_t^*\left[\frac{1/P_{t-1}^{M^*}}{P_{t-1}^{M^*}/P_{t-2}^{M^*}}\right]
$$
$$
+E_t D_{t,t+1}^{pa}\left\{
\begin{array}{c}
\phi^{M^*}\left[\frac{P_{t+1}^{M^*}/P_t^{M^*}}{P_t^{M^*}/P_{t-1}^{M^*}} - 1\right] \times \\
P_{t+1}^{M^*} S_{t+1} M_{t+1}^*\left(\frac{1}{P_t^{M^*}}\right)^2\left(\frac{P_{t+1}^{M^*}}{P_t^{M^*}/P_{t-1}^{M^*}}\right)
\end{array}
\right\} = 0.
\tag{28}
$$

11

2.4 Households

There are two types of households in the home economy, 'impatient' and 'patient'. Impatient households are characterized by a lower discount factor than patient households. The impatient households, comprising a share equal to $\varrho \in [0, 1\rangle$, are credit constrained and may only borrow up to a fraction, the loan-to-value ratio, of the market value of their housing stock. As in Iacoviello (2005), we assume that this constraint is always binding. The remaining $1 - \varrho$ share of households, the patient households, have unconstrained access to capital markets both home and abroad. Moreover, patient households own all domestic firms, including banks, and receive all dividends.

Each household supplies a differentiated labor input to intermediate firms. Wages are set under the assumption of monopolistic competition. We allow wages to differ between the two types of households.

Households obtain utility from housing services, consumption of other goods and services and leisure. Housing services are proportional to the housing stock. The stock of housing is supplied by an additional production sector, which simply takes the final good as input. Productivity growth in the production of housing is assumed to be lower than in the rest of the economy, which is consistent with the observed trend in the relative price of housing.

Preferences are additively separable in consumption and labor. Letting $U_t^k(j)$ denote the lifetime expected utility of a representative household j of type $k = im, pa$, we have:

$$U_t^k(j) = E_t \sum_{i=0}^{\infty} \left(\beta^k\right)^i \left[z_{t+i}^u u\left(C_{t+i}^k(j)\right) + z_{t+i}^h \omega(H_{t+i}^k(j)) - v\left(l_{t+i}^k(j)\right) \right], \quad (29)$$

where C_t^k denotes consumption, l_t^k is labor, and H_t^k is housing services. Households are assumed to live infinitely and they discount future utility by a discount factor $0 < \beta^k < 1$. As mentioned, the discount factor of impatient households, β^{im}, is assumed to be lower than the one of patient households, β^{pa}. We include a random taste shifter, z_t^h, to allow for shocks to housing preferences and, similarly, z_t^u for consumption of other

goods and services. They evolve according to:

$$\ln\left(\frac{z_t^u}{z^u}\right) = \lambda^u \ln\left(\frac{z_{t-1}^u}{z^u}\right) + \varepsilon_t^u, \qquad 0 \le \lambda^u < 1, \quad \varepsilon_t^u \sim iid\left(0, \sigma_u^2\right), \qquad (30)$$

$$\ln\left(\frac{z_t^h}{z^h}\right) = \lambda^h \ln\left(\frac{z_{t-1}^h}{z^h}\right) + \varepsilon_t^h, \qquad 0 \le \lambda^h < 1, \quad \varepsilon_t^h \sim iid\left(0, \sigma_h^2\right),$$

The current period utility functions for consumption, labor choices, and housing services, $u(C_t^k(j))$, $v(l_t^k(j))$, and $\omega(H_{t+i}^k(j))$ are given by:

$$u\left(C_t^k(j)\right) = (1 - b^c/\pi^z) \ln\left[\frac{C_t^k(j) - b^c C_{t-1}^k}{1 - b^c/\pi^z}\right],$$

$$v\left(l_t^k(j)\right) = \frac{1 - b^l}{1 + \zeta}\left[\frac{l_t^k(j) - b^l l_{t-1}^k}{1 - b^l}\right]^{1+\zeta}. \qquad (31)$$

$$\omega(H_t^k(j)) = \left(1 - b^h \pi^h/\pi^z\right) \ln\left[\frac{H_t^k(j) - b_h H_{t-1}^k}{1 - b^h \pi^h/\pi^z}\right]$$

We assume external habit persistence in consumption, housing and labor. The degrees of habit are governed by the parameters b^c $(0 < b^c < 1)$, b^h $(0 < b^h < 1)$ and b^l $(0 < b^h < 1)$, respectively. Thus, what generates utility is not only how much household j consumes today, but also how much it consumes relative to aggregate consumption in the previous period. This type of habit persistence is sometimes referred to as "keeping up with the Joneses". The motivation for this kind of utility is primarily to generate some sluggishness in consumption in response to shocks, which is consistent with stylized facts.[9] The degree of disutility of supplying labor is captured by the parameter $\zeta > 0$. The log-utility specification for consumption is chosen to ensure a balanced growth path.[10]

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[9]The specific functional form of the subutility function, $u(j)$, adapted here ensures that the habit parameter does not enter the steady state solution of the model.

[10]This is equivalent to letting $\sigma \to 1$ in the more general specification

$$u_t(C_t^k(j)) = \frac{\left(C_t^k(j) - b^c C_{t-1}^k\right)^{1-\sigma}}{1 - \sigma}$$

13

Each household is the monopolistic supplier of a labor input j (i.e. possesses a particular variety of labor, which it offers to firms), for which it sets the nominal wage, $W_t^k(j)$, taking into account the demand for labor from firms in the intermediate sector, given by (20). Following Kim (2000), there is sluggish wage adjustment due to resource costs that are measured in terms of the total wage bill. The adjustment costs, $\gamma_t^{W,k}$, are specified as:

$$\gamma_t^{W,k}(j) \equiv \frac{\phi^W}{2} \left[\frac{W_t^k(j)/W_{t-1}^k(j)}{W_{t-1}^k/W_{t-2}^k} - 1 \right]^2, \tag{32}$$

where W_t^k is the aggregate nominal wage rate of type k. As can be seen from (32), costs are related to changes in wage inflation relative to the past observed rate for households of type k. The parameter $\phi^W > 0$ determines how costly it is to change the wage inflation rate.

Patient households The individual flow budget constraint for the j patient households is:

$$P_t C_t^{pa}(j) + B_t^{pa}(j) + S_t B_{H,t}^*(j) + P_t^H H_t^{pa}(j) \leq W_t^{pa}(j) l_t^{pa}(j) \left[1 - \gamma_t^{W,pa}(j) \right]$$

$$+ \left[1 - \gamma_{t-1}^{B^*} \right] \left(1 + r_{t-1}^* \right) S_t B_{H,t-1}^*(j) \tag{33}$$

$$+ \left(1 + r_{t-1}^d \right) B_{t-1}^{pa}(j) + P_t^H (1 - \delta_H) H_{t-1}^{pa}(j) + \frac{1}{1-\varrho} \Psi_t(j) - \frac{1}{1-\varrho} \Xi_t(j),$$

where $B_t^{pa}(j)$ is patient household j's end of period t deposits, $B_{H,t}^*(j)$ is end of period t portfolio of foreign bonds (held by domestic households), P_t^H the nominal house prices, and H_t^{pa} housing. Furthermore, the domestic net short-term nominal interest rate on deposit is denoted by r_t^d, and the net nominal return on foreign bonds is r_t^*. The variable Ψ_t includes all profits and also any nominal adjustment costs, which are rebated in a lump-sum fashion. Finally, home agents pay lump-sum (non-distortionary) net taxes,

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where σ is the inverse of the intertemporal elasticity of substitution.

Ξ_t, denominated in home currency.[11]

A financial friction, γ^{B^*}, is introduced to guarantee that the net asset positions follow a stationary process.[12] This cost depends on the average net foreign asset position (detrended) of the domestic economy relative to some desired net foreign asset position, $\overline{B}_t^{H^*}$ (which may deviate from zero). Specifically, we adopt the following functional form:

$$\gamma_t^{B^*} = \phi^{B1} \frac{\exp\left(\phi^{B2}\left((1-\varrho)\frac{S_t}{P_t Z_t}\left(B_t^{H^*} - \overline{B}_t^{H^*}\right)\right)\right) - 1}{\exp\left(\phi^{B2}\left((1-\varrho)\frac{S_t}{P_t Z_t}\left(B_t^{H^*} - \overline{B}_t^{H^*}\right)\right)\right) + 1} + z_t^B, \tag{34}$$

where $0 \le \phi^{B1} \le 1$, $\phi^{B2} > 0$ and $B_t^{H^*} \equiv \left(\frac{1}{(1-\varrho)}\right) \int_0^{(1-\varrho)} B_{H,t}^{*,pa}(j) dj$ defines the home country's holdings of foreign bonds per patient household. The variable z_t^B can be interpreted as a risk premium shock and follows:

$$z_t^B = \lambda^B z_{t-1}^B + \varepsilon_t^B. \tag{35}$$

Households choose consumption, housing, labor, deposits, foreign bond holdings, and wages to maximize the discounted utility given by (29), taking into account the budget restriction (33) and the demand for labor (20).

The intertemporal optimality conditions are given by (skipping index):

$$\frac{1}{1+r_t^d} = E_t D_{t,t+1}^{pa} \tag{36}$$

$$E_t\left(D_{t,t+1}^{pa} \frac{S_{t+1}}{S_t}\right) = \frac{\left(1+r_t^d\right) E_t D_{t,t+1}^{pa}}{\left(1+r_t^*\right)\left[1-\gamma_t^{B^*}\right]}, \tag{37}$$

where the stochastic discount factor, $D_{t,t+1}^{pa}$, is defined as:

$$D_{t,t+1}^{pa} = \beta^{pa} \frac{U_{C_{t+1}}^{pa\prime}}{U_{C_t}^{pa\prime}} \frac{P_t}{P_{t+1}} = \beta^{pa} \frac{P_t}{P_{t+1}} \frac{z_{t+1}^u}{z_t^u} \frac{C_t^{pa} - b^c C_{t-1}^{pa}}{C_{t+1}^{pa} - b^c C_t^{pa}}. \tag{38}$$

---

[11]Since it is assumed that intermediate firms are owned by savers, they all receive a share $\frac{1}{(1-\varrho)}$ of per capita dividends. Furthermore, only savers pay tax.

[12]See Schmitt-Grohe and Uribe (2003) for a discussion and for alternative ways to ensure stationarity.

Equation (36) is the consumption Euler equation for the savers. It states that along an optimal consumption path the marginal rate of substitution between consumption tomorrow and consumption today must equal the gross real interest rate. If this does not hold, utility could be increased by reallocating resources across time. Equation (37) is a version of the Uncovered Interest Parity (UIP). It summarizes the optimal holdings of domestic and international bonds. In equilibrium, it should not be possible to increase the portfolio return by changing the composition of domestic and foreign bonds.

The first-order condition for wage setting reads:

$$
\frac{W_t^{pa}}{P_t} = \psi_t \Phi_t^{pa} \left[\begin{array}{c} \left(1 - \gamma_t^{W,pa}\right) \\ + \frac{\phi^W W_t^{pa}/W_{t-1}^{pa}}{W_{t-1}/W_{t-2}} \left(\frac{W_t^{pa} W_{t-1}^{pa}}{W_{t-1}/W_{t-2}} - 1 \right) \\ -E_t \left\{ \begin{array}{c} D_{t,t+1}^{pa} \pi_{t+1}^W \times \\ \frac{l_{t+1}^{pa}}{l_t^{pa}} \frac{\phi^W W_{t+1}^{pa}/W_t^{pa}}{W_t^{pa}/W_{t-1}^{pa}} \left(\frac{W_{t+1}^{pa}/W_t^{pa}}{W_t^{pa}/W_{t-1}^{pa}} - 1 \right) \end{array} \right\} \end{array} \right]^{-1}, \tag{39}
$$

where Φ_t^{pa} measures the savers' marginal rate of substitution of consumption for leisure:

$$
\Phi_t^{pa} = -\frac{U_{L_t}^{pa\prime}}{U_{C_t}^{pa\prime}} = \frac{1}{z_t^u} \frac{C_t^{pa} - b^c C_{t-1}^{pa}}{1 - b^c/\pi^z} \left(\frac{L_t^{pa} - b^l L_{t-1}^{pa}}{1 - b^l} \right)^\zeta. \tag{40}
$$

When setting wages, households balance their disutility from working and their utility of consumption generated from their labor income. The optimal real wage is set as a markup over Φ_t^{pa}. The markup depends on how much market power households have in the labor market, governed by the time-varying parameter ψ_t (the elasticity of substitution between differentiated labor types). Hence, the size of ψ_t could be interpreted as the bargaining power of the households (or labor unions) in the wage setting process. The total markup also depends on the costs of adjusting wages.

The first-order condition for the demand for housing is

$$
\frac{z_t^h}{z_t^u} \frac{1 - b^h \pi^h/\pi^z}{1 - b^c/\pi^z} \frac{C_t^{pa} - b^c C_{t-1}^{pa}}{H_t^k - b_h H_{t-1}^k} = P_t^H - E_t \left[D_{t,t+1}^{pa} (1 - \delta_H) P_{t+1}^H \right]
$$

16

In order to derive the demand for deposits we make the assumption that deposits contracts are a composite constant elasticity of substitution basket of slightly differentiated products with elasticity term equal to $\theta_t^d < -1$

$$B_t^{pa}(j) = \left[\int_0^1 B_t^{pa}(j,i)^{(\theta_t^d-1)/\theta_t^d}\, di \right]^{\frac{\theta_t^d}{(\theta_t^d-1)}}$$

Demand for deposits from households j to bank i is obtained by maximizing the revenue of total savings

$$\int_0^1 r_t^d(i)\, B_t^{pa}(j,i)\, di$$

with respect to $B_t^{pa}(j,i)$, subject to $\left[\int_0^1 B_t^{pa}(j,i)^{(\theta_t^d-1)/\theta_t^d}\, di \right]^{\theta_t^d/(\theta_t^d-1)} \leq B_t^{pa}(j)$, where $B_t^{pa}(j)$ is the overall amount of deposits of household j. The resulting aggregate demand for deposits in bank i is given by

$$B_t^{pa}(i) = \left(\frac{r_t^d(i)}{r_t^d} \right)^{-\theta_t^d} B_t^{pa} \tag{41}$$

where B_t^{pa} is the aggregate deposit in the economy and $r_t^d = \left[\int_0^1 r_t^d(i)^{1-\theta_t^d}\, di \right]^{1/(1-\theta_t^d)}$ is the aggregate deposit rate.

The stochastic elasticity evolves as follows

$$\ln\left(\theta_t^d\right) = \left(1 - \lambda^{\theta^d}\right) \ln\left(\theta^d\right) + \lambda^{\theta^d} \ln\left(\theta_{t-1}^d\right) + \varepsilon_t^{\theta^d}, \qquad 0 \leq \lambda^{\theta^d} < 1, \quad \varepsilon_t^{\theta^d} \sim iid\left(0, \sigma_{\theta^d}^2\right)$$

Impatient households Impatient households face a similar problem. However, their budget constraint reflects the fact that they do not have access to international bonds markets or the possibility of investing in domestic firms. Hence, for impatient house-

17

holds we have the following budget constraint

$$P_t C_t^{im}(j) + \left(1 + r_{t-1}^{im}\right) B_{t-1}^{im}(j) + P_t^H H_t^{im}(j) \le W_t^{im} l_t^{im}(j) \left[1 - \gamma_t^{W^{im}}(j)\right]$$
$$+ B_t^{im}(j) + P_t^H \left(1 - \delta_H\right) H_{t-1}^{im}(j) \tag{42}$$

where $B_t^{im}(j) > 0$ now denotes the amount borrowed and r_t^{im} is the nominal net interest rate paid on borrowing (which can be interpreted as the mortgage rate). The remaining variables have an identical interpretation to the patient household case.

Impatient household borrowing is restricted to a fraction of the expected value of their collateral, namely their housing stock. This constraint can be formulated as:

$$\left(1 + r_t^{im}\right) B_t^{im}(j) \le \Omega_t E_t \left[P_{t+1}^H H(j)_t^{im}\right] \tag{43}$$

where Ω_t is the loan-to-value ratio assumed to follow an autoregressive process

$$\ln\left(\frac{\Omega_t}{\Omega}\right) = \lambda^\Omega \ln\left(\frac{\Omega_{t-1}}{\Omega}\right) + \varepsilon_t^\Omega, \qquad 0 \le \lambda^\Omega < 1, \quad \varepsilon_t^\Omega \sim iid\left(0, \sigma_\Omega^2\right),$$

Impatient households maximize utility given the same set of constraints as patient households. In addition, impatient households take into account the collateral constraint. The first order conditions can be summarized as follows (in symmetric equilibrium):

$$\frac{U_{H_t}^{im\prime}}{U_{C_t}^{im\prime}} = P_t^H - E_t \left[D_{t,t+1}^{im}(1 - \delta_H)P_{t+1}^H - \left(\frac{1}{1 + r_{t-1}^{b,im}} - D_{t,t+1}^{im}\right)\Omega_t P_{t+1}^H\right] \tag{44}$$

where
$$U_{C_t}^{im\prime} = z_t^u \left(\frac{C_t^{im} - b^c C_{t-1}^{im}}{1 - b^c/\pi^z}\right)^{-1}$$

and
$$U_{H_t}^{im\prime} = z_t^h \left(\frac{H_t^{im} - b^h H_{t-1}^{im}}{1 - b^h/\pi^z}\right)^{-1}$$

18

and

$$D_{t,t+1}^{im} = \beta^{im} \frac{U_{C_{t+1}}^{im\prime}}{U_{C_t}^{im\prime}} \frac{P_t}{P_{t+1}}$$

is the sthocastic discount factor.

Furthermore,

$$\frac{W_t^{im}}{P_t} = \psi_t \Phi_t^{im} \left[\begin{array}{c} \left(1 - \gamma_t^{W,im}\right) \\ + \frac{\phi^W W_t^{im}/W_{t-1}^{im}}{W_{t-1}^{im}/W_{t-2}^{im}} \left(\frac{W_t^{im} W_{t-1}^{im}}{W_{t-1}^{im}/W_{t-2}^{im}} - 1\right) \\ -E_t \left\{ \begin{array}{c} D_{t,t+1}^{im} \pi_{t+1}^W \times \\ \frac{l_{t+1}^{im}}{l_t^{im}} \frac{\phi^W W_{t+1}^{im}/W_t^{im}}{W_t^{im}/W_{t-1}^{im}} \left(\frac{W_{t+1}^{im}/W_t^{im}}{W_t^{pa}/W_{t-1}^{im}} - 1\right) \end{array} \right\} \end{array} \right]^{-1}, \tag{45}$$

where Φ_t^{im} measures the savers' marginal rate of substitution of consumption for leisure, defined as follows

$$\Phi_t^{im} = -\frac{U_{L_t}^{im\prime}}{U_{C_t}^{im\prime}}$$

where

$$U_{L_t}^{im\prime} = \left(\frac{L_t^{im} - b^l L_{t-1}^{im}}{1 - b^l}\right)^{\zeta}$$

The demand for mortgages is derived assuming that loans are a composite constant elasticity of substitution basket of differentiated financial products with elasticity denoted by $\theta_t^{b,im} > 1$. In particular, borrowing by household j obey the following Dixit-Stiglitz index:

$$B_t^{im}(j) = \left[\frac{1}{\varrho} \int_0^{\varrho} B_t^{im}(j,i)^{(\theta_t^{b,im}-1)/\theta_t^{b,im}} di\right]^{\frac{\theta_t^{b,im}}{(\theta_t^{b,im}-1)}} \tag{46}$$

This assumption is key to allow for the existence of a positive mark-up (spread) of the mortgage rate over the wholesale (money market) rate.

The optimal combination of credit from different banks is found by minimizing the total repayment $\int_0^1 r_t^{b,im}(i) B_t^{im}(j,i) di$ subject to (46), which yields the following

demand schedule for mortgages

$$B_t^{im}(i) = \left(\frac{r_t^{b,im}(i)}{r_t^{b,im}}\right)^{-\theta_t^{b,im}} B_t^{im} \tag{47}$$

where B_t^{im} is the overall volume of mortgages and $r_t^{b,im} = \left[\int_0^1 r_t^{b,im}(i)^{1-\theta_t^{b,im}} di\right]^{1/\left(1-\theta_t^{b,im}\right)}$ is the mortgage rate index. The elasticity of substitution, $\theta_t^{b,im}$, is stochastic and evolves according to

$$\ln\left(\theta_t^{b,im}\right) = \left(1 - \lambda^{\theta^{b,im}}\right) \ln\left(\theta^{b,im}\right) + \lambda^{\theta b,im} \ln\left(\theta_{t-1}^{b,im}\right) + \varepsilon_t^{\theta^{b,im}}, \ \ 0 \le \lambda^{\theta^{b,im}} < 1, \ \ \varepsilon_t^{\theta^{b,im}} \sim iid\left(0, \sigma_{\theta^{b,im}}^2\right)$$

2.5 Banking sector

Banks are assumed to operate under monopolistic competition, and are restricted by their balance sheet identity, stating that lending must equal deposits plus bank capital. Bank capital is accumulated through retained earnings and can only be adjusted gradually. Furthermore, borrowing from Benes and Kumhof (2011), we assume that banks have to adhere to a regulatory capital requirement. Failing to do so, will incur a cost proportional to total assets (lending). The existence of an idiosyncratic shock to returns, will typically lead banks to aim for a cushion above the required rate. Bank capital plays an important role for credit supply in the model through a potential feedback loop between the real and the financial side of the economy. For example, an economic downturn could possibly hit bank profits and reduce the bank capital ratio, with banks cutting back on lending as a consequence. This would in turn give a further negative impetus to the real economy.

As suggested by Gerali et al. (2010), we can think of each bank as composed of two "retail" branches and a "wholesale" branch. One retail branch is responsible for providing differentiated loans to households and to entrepreneurs, while the other retail branch takes care of the deposit side. Both branches set interest rates in a monopolistically competitive fashion, subject to adjustment costs. The wholesale branch manages the

capital position of the bank. Its task is to choose the overall level of operations regarding deposit and lending, taking into account the capital requirement, and internalizing the distribution of the idiosyncratic shock to overall returns.

The maximization problem of banks is affected by the nature of financial frictions which firms and households are subject to. In particular, loans from firms is facing Bernanke et al. (1999) type of frictions. This implies that banks account for the possibility of entrepreneurs' default. Moreover, in the seminal BGG paper the lender operates in a perfectly competitive market. In our set-up banks are monopolisticly competitive. This requires to adjust the original BGG borrowing contract to take into account that feature. Hafstead and Smith (2012) provide a suitable framework.

2.5.1   Wholesale branch

Each wholesale branch operates under perfect competition. On the liability side, it combines net worth, or bank capital (K^B), and wholesale deposits (B^{pa}), while on the asset side, it issues loans (B). Thus, the balance sheet of bank $i \in [0, 1]$ is simply:

$$B_t^{pa}(i) + K_t^B(i) = B_t(i) \tag{48}$$

with

$$B_t(i) = B_t^e(i) + \varrho B_t^{im}(i)$$

Bank capital is accumulated retained profits, adjusted for dividend rate δ^B:

$$K_t^B(i) = \left(1 - \delta^B\right) K_t^B(i) + J_{t-1}^B(i)$$

where J_t^B denotes overall bank profits.

Banks differ in that the overall return on their lending is subject to an idiosyncratic

21

shock ω_t^B.[13] The capital requirement, γ_t^B, is implicitly defined by:

$$R_t^A(i) B_t(i) \omega_t^B(i) - R_t^d B_t^{pa}(i) < \gamma_t^B R_t^A(i) B_t(i) \omega_t^B(i) \tag{49}$$

and

$$R_t^A(i) = \varrho \frac{B_t^{im}(i)}{B_t(i)} R_t^{b,im}(i) + \frac{B_t^e(i)}{B_t(i)} R_t^e(i)$$

is the average return on banking sector activities. Failing to comply with the capital requirement will incur a penalty χB_t. For a given level of operations and returns, there will exist a level of the idiosyncratic shock, $\overline{\omega}_t^B$, such that whenever $\omega_t^B < \overline{\omega}_t^B$ banks will fail to meet the capital requirement. Using condition (49) we can define this cut-off value as:

$$\overline{\omega}_t^B(i) = \frac{R_t^d(i) B_t^{pa}(i)}{(1 - \gamma_t) R_t^A(i) B_t(i)}$$

Taking the gross wholesale lending rate, R_t^b, and the gross wholesale deposit rate, R_t^{wd}, as given, the problem for the wholesale bank is to choose loans and deposits to maximize expected profits, internalizing the costs related to breaching the capital requirement

$$\max_{\left\{B_t(i), B_t^{pa}(i)\right\}} E_t \left[R_t^b(i) B_t(i) - R_t^{wd}(i) B_t^{pa}(i) - \chi B_t(i) F\left(\overline{\omega}_t^B(i)\right) \right] \tag{50}$$

By defining $\iota_t^B = \frac{B_t}{K_t^B}$ we can re-write (50) as

$$\max_{\left\{\iota_t^B\right\}} E_t \left[\left(R_t^b(i) - R_t^{wd}(i) \right) \iota_t^B(i) + R_t^{wd}(i) - \chi \iota_t^B F\left(\frac{R_t^d(i)\left(1 - \frac{1}{\iota_t^B(i)}\right)}{(1 - \gamma_t) R_t^A(i)} \right) \right]$$

with first order condition

$$E_t \left\{ R_t^b(i) - R_t^{wd}(i) - \chi \left[F\left(\overline{\omega}_t^B(i)\right) + f\left(\overline{\omega}_t^B(i)\right) \overline{\omega}_t^B(i) \left(\iota_t^B(i) - 1\right) \right] \right\} = 0 \tag{51}$$

---

[13] We assume that ω_t^B is log-normally distributed with $E\left\{\omega_t^B\right\} = 1$, standard deviation σ_t^B, probability density function $f\left(\omega_t^B\right)$ and cumulative distribution $F\left(\omega_t^B\right)$.

As in Gerali et al. (2010), to close the model, we assume that banks have access to unlimited finance at the policy rate $R_t = 1 + r_t$ from a lending facility at the central bank, implying an arbitrage condition such that $R_t^{wd} = R_t$. Hence equation (51) states that the wholesale interest rate is equal to the policy rate plus a term which depends on the penalty coefficient χ and on expressions that determine the likelihood of a breach.[14]

We can define the wholesale spread S_t^w as

$$S_t^w (i) \equiv R_t^b (i) - R_t = \chi \left[F \left(\overline{\omega}_t^B (i) \right) + f \left(\overline{\omega}_t^B (i) \right) \overline{\omega}_t^B (i) \left(\iota_t^B (i) - 1 \right) \right] \qquad (52)$$

2.5.2 Deposit Branch

The retail deposit branch of bank i collects deposits $B_t^{pa} (i)$ from patient households and passes them to the wholesale unit which remunerates them at rate $R_t^{wd} = R_t$. We assume that there are quadratic adjustment costs related to changing the deposit rate. The deposit branch chooses the deposit rate R_t^d to maximize

$$\max_{\{R_t^d\}} E_t \sum_{s=0}^{\infty} D_{0,t+s}^{pa} \left[\begin{array}{c} R_{t+s} B_t^{pa} (i) - R_{t+s}^d (i) B_t^{pa} (i) \\ -\frac{\phi^d}{2} \left(\frac{R_t^d(i)}{R_{t-1+s}^d(i)} - 1 \right)^2 R_{t+s}^d B_t^{pa} (i) \end{array} \right]$$

subject to the demand for deposit (41). The first-order condition is

$$- \left(1 - \theta_t^d \right) - \theta_t^d \frac{R_t}{R_t^d} - \phi^d \left(\frac{R_t^d}{R_{t-1}^d} - 1 \right) \frac{R_t^d}{R_{t-1}^d} + E_t \left[D_{t,t+1}^{pa} \phi^d \left(\frac{R_{t+1}^d}{R_t^d} - 1 \right) \left(\frac{R_{t+1}^d}{R_t^d} \right)^2 \frac{B_t^{pa} (i)}{B_{t-1}^{pa} (i)} \right] = 0$$

When all rates are flexible ($\phi^d = 0$) the deposit rate R_t^d is a markdown over the policy rate, $R_t^d = \frac{\theta_t^d}{\theta_t^d - 1} R_t$.

2.5.3 Loan Branch

Banks lend to households and entrepreneurs. An underlying assumption is that there exist a problem of asymmetric information between lender and borrower. Furthermore,

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[14]This is the main difference with Gerali et al. (2010). In their set up the term depends only on banks leverage and on the cost associated to its variation away from an exogenously fixed target level.

it is costly for banks to monitor borrowers. For households this problem is solved by imposing the collateral constraint, implicitly assuming that creditors cannot force households to repay their debt unless the debts are secured by collateral. Alternatively, this framework is consistent with assuming that monitoring costs always exceed the value of the debt. Regarding entrepreneurs, monitoring costs are assumed to be a fraction, μ, of entrepreneurs total assets, $\overline{\phi}$ (defined in (61)). Entrepreneurs are identical up to an idiosyncratic productivity shock, ω. The optimal contract between banks and entrepreneurs, will imply that all entrepreneurs who draw $\omega < \overline{\omega}$ will declare bankruptcy and leave all its remaining assets net of monitoring costs to the bank. Letting $F(\cdot)$ denote the cumulative distribution of the idiosyncratic productivity shock, the probability of default is given by $F(\overline{\omega})$. The optimal contracting problem is discussed in the next section.

The loan branch of bank i obtains wholesale loans $B_t^w(i)$ at the unitary cost R_t^b, differentiates them at no cost and resells them to impatient households and entrepreneurs applying two different markups. It also faces a quadratic cost of adjustment in changing lending rates proportional to aggregate returns on loans.

The problem of the loan branch is to choose the mortgage rate $R_t^{im}(i)$ and the gross lending rate to entrepreneurs $R_t^{eg}(i)$ in order to maximize

$$\max_{\left\{R_{t+s}^{im}(i), R_{t+s}^{eg}(i)\right\}} E_t \sum_{s=0}^{\infty} D_{0,t+s}^{pa} \left[\begin{array}{c} R_{t+s}^{im}(i) B_{t+s}^{im}(i) + \left(1 - F\left(\overline{\omega}_{t+s}^e\right)\right) R_{t+s}^{eg}(i) B_{t+s}^e(i) \\ -R_{t+s}^b(i) B_{t+s}(i) - \frac{\phi^{im}}{2} \left(\frac{R_{t+s}^{im}(i)}{R_{t-1+s}^{im}(i)} - 1 \right)^2 R_{t+s}^{im}(i) B_{t+s}^{im}(i) \\ -\frac{\phi^e}{2} \left(\frac{R_{t+s}^{eg}(i)}{R_{t-1+s}^{eg}(i)} - 1 \right)^2 R_{t+s}^{eg}(i) B_{t+s}^e(i) \\ +\left(1 - \mu_t\right) \overline{\phi}_t \frac{B_{t+s}^e(i)}{B_{t+s}^e} \end{array} \right]$$

subject to the demand for loans (57) and (47), and $B_t(i) = B_t^{im}(i) + B_t^e(i)$.

24

Households The first-order condition for households results in (after imposing a symmetric equilibrium):

$$1-\theta_t^{im}+\theta_t^{im}\frac{R_t^b}{R_t^{im}}-\phi^{im}\left(\frac{R_t^{im}}{R_{t-1}^{im}}-1\right)\frac{R_t^{im}}{R_{t-1}^{im}}+E_t\left[D_{t,t+1}^{pa}\phi^{im}\left(\frac{R_{t+1}^{im}}{R_t^{im}}-1\right)\left(\frac{R_{t+1}^{im}}{R_t^{im}}\right)^2\frac{B_{t+1}^{im}}{B_t^{im}}\right]=0 \tag{53}$$

This expression has several implications. First, it states that the mortgage rate is set based on current and expected future values of markup shocks and of the wholesale rate (the relevant measure of marginal cost), which in turn depends on the policy rate and on the capital position of the bank. Second, it is possible to show that the spread on mortgages S_t^{im} is increasing in the policy rate, and it is proportional to the wholesale spread S_t^w. In fact, when rates are perfectly flexible ($\phi^{im}=0$), (53) reduces to

$$R_t^{im}=\frac{\theta_t^{b,im}-1}{\theta_t^{b,im}}R_t^b$$

Using expression (52)
$$R_t^{im}=\frac{\theta_t^{im}-1}{\theta_t^{im}}\{S_t^w+R_t\}$$

$$S_t^{im}\equiv R_t^{im}-R_t=\frac{\theta_t^{im}}{\theta_t^{im}-1}S_t^w+\frac{1}{\theta_t^{im}-1}R_t$$

Entrepreneurs The first-order condition for entrepreneurs has similar features and is given by:

$$-\theta^e\frac{\overline{R}_t^e}{R_t^{eg}}+\theta^e\frac{R_t^b}{R_t^N}-\phi^u\left(\frac{R_t^{eg}}{R_{t-1}^{eg}}-1\right)\frac{R_t^{eg}}{R_{t-1}^{eg}}+E_t\left[D_{t,t+1}^{pa}\phi^e\left(\frac{R_{t+1}^{eg}}{R_t^{eg}}-1\right)\left(\frac{R_{t+1}^{eg}}{R_t^{eg}}\right)^2\frac{B_{t+1}^e}{B_t^e}\right]=0$$

where
$$\overline{R}_t^e\equiv\frac{\theta^e-1}{\theta^e}\left[1-F_t\left(\overline{\omega}_t^e\right)\right]R_t^{eg}+\frac{(1-\mu)\,\overline{\phi}_t}{B_t^e} \tag{54}$$

represents net revenues from a marginal change in the supply of credit. Marginal net revenues are equated to marginal costs, which are determined by the funding rate and

the adjustment costs. Equation (54) is required to hold and will act as a restriction in the contracting problem to be discussed in the next section.

Finally we can define overall bank profits as follows

$$J_t^B(i) = r_t^{im} \varrho B_t^{im}(i) + [R_t^e(i) - 1] B_t^e(i) - [R_t^d - 1] B_t^{pa}(i) - \chi F\left(\overline{\omega}_t^B(i)\right) B_t(i) \quad (55)$$

where $R_t^e(i)$ and $B_t^e(i)$ refer to Entrepreneurs described in the next section.

2.6   Entrepreneurs

There is a continuum of Entrepreneurs defined on $s \in [0,1]$ who rent capital to firms. Capital goods are obtained from the capital producing sector. To finance their purchases of capital goods entrepreneurs can count on internal funds N_t^e, but in addition, they depend on external funding from banks B_t^e. The entrepreneurs' balance sheet in nominal terms can be defined as

$$B_t^e(s) + N_t^e(s) = P_t^K K_t(s) \quad (56)$$

Entrepreneurs are identical up to an idiosyncratic i.i.d. productivity shock ω_t^e.[15] Hence, ex-post aggregate returns will vary across entrepreneurs. Entrepreneurs start each period by negotiating a contract with the banks, specifying both the interest rate to pay, R_t^{eg}, and the borrowing amount, B^e. Using a standard Dixit-Stiglitz aggregator to describe the monopolistic competition, it is possible to define aggregate loans and gross nominal interest rate as follows

$$B_t^e(s) = \left[\int_0^1 B_t^e(s)^{\frac{\theta^e - 1}{\theta^e}} ds \right]^{\frac{\theta^e}{\theta^e - 1}}$$

---

[15] It is assumed to have a cumulative distribution $F_t(\omega_t^e)$ such that $F_t(x_t) = \Pr[\omega_t^e \le x_t]$ and a probability distribution function $f_t(\omega_t^e)$. It is log-normally distributed with mean equal to 1 and standard deviation σ_t^e. As in Christiano et al. (2014) σ_t^e is represented by the following process

$$\ln\left(\frac{\sigma_t^e}{\sigma^e}\right) = \lambda^{\sigma^e} \ln\left(\frac{\sigma_{t-1}^e}{\sigma^e}\right) + \varepsilon_t^{\sigma^e}, \qquad 0 \le \lambda^{\sigma^e} < 1, \quad \varepsilon_t^{\sigma^e} \sim iid\left(0, \sigma_{\sigma^e}^2\right)$$

and it is defined as risk shock capturing movements in the riskiness of borrowers.

$$R_t^{eg}(s) = \left[\int_0^1 R_t^{eg}(s)^{1-\theta^e} \, ds \right]^{\frac{1}{1-\theta^e}}$$

where θ^e is the elasticity of substitution among loans given by the assumption that loans are a composite constant elasticity of substitution basket of slightly differentiated products It follows that each bank i faces a downward sloping demand curve for loans given by

$$B_t^e(s) = \left(\frac{R_t^{eg}(s)}{R_t^{eg}} \right)^{-\theta^e} B_t^e \tag{57}$$

Capital expenditures, and therefore external borrowing, are chosen before the realization of the idiosyncratic shock. After the realization of the productivity shock, some entrepreneurs will find themselves unable to repay their debt in full. Entrepreneurs who do not have the resources to repay their loans declare bankruptcy and banks receive the assets of bankrupt entrepreneurs minus the monitoring cost. For simplicity, we model the monitoring cost as a constant proportion μ of the realized gross real return to entrepreneurs, $R_{t+1}^{ke} P_t^K K_t$ where the return to capital is defined in (68). Entrepreneurs who declare bankruptcy are characterized by a productivity level $\omega^e < \overline{\omega}^e$, where the cut-off value $\overline{\omega}^e$ satisfies:

$$\overline{\omega}_t^e(s) R_t^{ke}(s) P_t^K K_t(s) = R_t^{eg}(s) B_t^e(s) \tag{58}$$

The optimal contract establishes a pair (R_t^{eg}, B_t^e) that satisfies the maximization of entrepreneurs' expected profits:

$$E_t \left[1 - \Gamma_t \left(\overline{\omega}_{t+1}^e(s) \right) \right] R_t^{ke} \frac{P_t^K K_t(s)}{N_t^e(s)}$$

subject to the lender's participation constraint, which states that the lender must be ensured a rate of return, R_t^e, defined implicitly by:

$$R_t^e(s) B_t^e(s) = \left[1 - F_t \left(\overline{\omega}_t^e(s) \right) \right] R_t^{eg}(s) B_t^e(s) + (1 - \mu) \overline{\phi}_t(s) \tag{59}$$

where $\Gamma_t \left(\overline{\omega}_{t+1}^e \right)$ is the share of profits going to the lender, defined by:

$$\Gamma_t \left(\overline{\omega}_t^e \right) = \int_0^{\overline{\omega}_t^e} \omega^e F_t \left(\omega^e \right) d\omega^e + \overline{\omega}_t^e \int_{\overline{\omega}_t^e}^\infty F_t \left(\omega^e \right) d\omega^e$$

The participation constraint has the same role as the zero profit condition in BGG. The required return must be on the banks pricing schedule. Hence by making use of equation (54), we get:

$$R_t^e(s) = \frac{\theta^e}{\theta^e - 1} \overline{R}_t^e(s) - \frac{(1 - \mu) \overline{\phi}_t}{(\theta^e - 1) B_t^e} \tag{60}$$

where

$$\overline{\phi}_t = \left[\int_0^{\overline{\omega}_t^e} \omega^e dF_t \left(\omega^e \right) \right] R_t^{ke} P_t^K K_t, \tag{61}$$

Hence, the required return is given as a mark-up over marginal lending costs minus a correction term, which reflects the fact that banks set mark-ups based on gross interest rates and not on assets received through bankruptcy. We can now reformulate the participation constraint in terms of the cut-off rate and the leverage ratio as follows:

$$\left[\Gamma_t \left(\overline{\omega}_t^e(s) \right) - \mu G_t \left(\overline{\omega}_t^e(s) \right) \right] R_t^{ke}(s) \frac{P_t^K K_t(s)}{N_t^e(s)} - R_t^e \left(\frac{P_t^K K_t(s)}{N_t^e(s)} - 1 \right) = 0 \tag{62}$$

with:

$$G_t \left(\overline{\omega}_t^e \right) = \int_0^{\overline{\omega}_t^e} \omega^e dF_t \left(\omega^e \right)$$

The first-order conditions with respect to $\overline{\omega}_t^e$ and $\frac{P_t^K K_t}{N_t^e}$ can be combined in the following efficiency condition

$$E_t \left\{ \left[1 - \Gamma_t \left(\overline{\omega}_{t+1}^e \right) \right] \frac{R_{t+1}^{ke}}{R_t^e} + \frac{\Gamma_t' \left(\overline{\omega}_{t+1}^e \right)}{\Gamma_t' \left(\overline{\omega}_{t+1}^e \right) - \mu G_t' \left(\overline{\omega}_{t+1}^e \right)} \left[\frac{R_{t+1}^{ke}}{R_t^e} \left(\Gamma_t \left(\overline{\omega}_{t+1}^e \right) - \mu G_t \left(\overline{\omega}_{t+1}^e \right) \right) - 1 \right] \right\} = 0 \tag{63}$$

Together with (62), we obtain two equations to solve for $\overline{\omega}_t^e$ and $\frac{P_t^K K_t}{N_t^e}$ as functions of

$\frac{R_{t+1}^{ke}}{R_t^e}$. In particular, we can derive the result that the external finance premium

$$S_t \equiv E_t \left\{ \frac{R_{t+1}^{ke}}{R_t^e} \right\}$$

is an increasing function of the entrepreneurs' leverage ratio $\frac{P_t^K K_t}{N_t^e}$. The higher the leverage ratio, the riskier the entrepreneur is and the higher the premium he has to pay on his external finance.

Net worth is cumulated out of retained profits net of the expected monitoring costs. To avoid that entrepreneurs reach a state where they can fully self-finance, it is assumed that each period entrepreneurs exit ("die") with a given probability γ_t^e.[16] Hence, the evolution of net worth can be expressed as follows:

$$N_t^e = \gamma_t^e \left[\left(R_t^{ke} - R_{t-1}^e - \mu G_t \left(\overline{\omega}_t^e \right) R_t^{ke} \right) P_{t-1}^K K_{t-1} + R_{t-1}^e N_{t-1}^e \right] + W^e$$

where W^e is an endowment received by entering entrepreneurs.

2.7 Capital goods producers

It is assumed that capital goods are produced by a separate sector in a competitive market. At the beginning of period t capital producers buy the depreciated physical capital stock $(1-\delta)K_{t-1}$ from the entrepreneurs and final goods I_t from retailers and convert them into capital stock K_t which is sold to entrepreneurs and used for production at date t. Capital production technology is characterized by convex capital adjustment costs and it is described by the following equation

$$K_t(h) = (1 - \delta) K_{t-1}(h) + \kappa_t(h) K_{t-1}(h), \tag{64}$$

---

[16]Following Christiano et al. (2014) we assume that this is a disturbance that directly affects the quantity of net worth in the hands of entrepreneurs. We call it equity shock and it follows the process

$$\ln \left(\frac{\gamma_t^e}{\gamma^e} \right) = \lambda^{\gamma^e} \ln \left(\frac{\gamma_{t-1}^e}{\gamma^e} \right) + \varepsilon_t^{\gamma^e}, \qquad 0 \leq \lambda^{\gamma^e} < 1, \quad \varepsilon_t^{\gamma^e} \sim iid \left(0, \sigma_{\gamma^e}^2 \right)$$

where $\delta \in [0,1]$ is the depreciation. Convex adjustment costs are described by function $\kappa_t (h)$, which measures the rate of capital accumulation. It is given by

$$
\begin{aligned}
\kappa_t (h) \;=\; & \frac{I_t (h)}{K_{t-1} (h)} - \frac{\phi^{I1}}{2} \left[\left(\frac{I_t (h)}{K_{t-1} (h)} - \frac{I}{K} \right) \right]^2 \\
& - \frac{\phi^{I2}}{2} \left(\frac{I_t (h)}{K_{t-1} (h)} - \frac{I_{t-1}}{K_{t-2}} \right)^2 ,
\end{aligned}
\tag{65}
$$

where I_t denotes investment and z_t^I is an investment shock.[17] The parameters ϕ^{I1} and ϕ^{I2} determine the cost of deviating from the steady state investment to capital ratio and the cost of changing this ratio, respectively.

The representative capital producer maximizes its future discounted profit stream taking the price of capital P_t^k as given. The first-order condition associated with it is

$$
Q_t^u(h) = \frac{1}{\kappa_t'(h)}
\tag{66}
$$

where $Q_t^u(h) \equiv \frac{P_t^k(h)}{P_t(h)}$ is the real price of capital and

$$
\kappa_t'(h) = 1 - \phi^{I1} \left(\frac{I_t(h)}{K_{t-1}(h)} - (\pi^z - 1 + \delta)\, Z_t^I \right) - \phi^{I2} \left(\frac{I_t(h)}{K_{t-1}(h)} - \frac{I_{t-1}}{K_{t-2}} \right),
\tag{67}
$$

We can finally define the return to capital as

$$
R_t^{ke} = \frac{R_t^K + (1 - \delta)\, P_t^K}{P_{t-1}^K}
\tag{68}
$$

2.8 Housing production sector

Residential investment is supplied by a continuum of perfectly competitive housing producers using the following representative technology

$$
IH_t = \kappa^H \left(\frac{I_t^H}{H_{t-1}} \right) H_{t-1}
\tag{69}
$$

---

[17]This shock could e.g. represent changes in the relative price of consumption and investment.

where IH_t denotes investment in new housing, I_t^H is the input of the final good used to produce new housing and the function $\kappa^H(\cdot)$ is given by :

$$
\kappa^H\left(\frac{I_t^H}{H_{t-1}}\right) = \frac{I_t^H}{H_{t-1}} - \frac{\phi_1^H}{2}\left[\left(\frac{I_t^H}{H_{t-1}} - (\delta_H - 1)z_t^H\right)\right]^2 \\
- \frac{\phi_2^H}{2}\left(\frac{I_t^H}{H_{t-1}} - \frac{I_{t-1}^H}{H_{t-2}}\right)^2 ,
\tag{70}
$$

where $\phi_1^H, \phi_2^H > 0$ are parameters and z_t^H is a housing investment specific shock following the process

$$
\ln\left(\frac{z_t^H}{z^H}\right) = \lambda^H \ln\left(\frac{z_{t-1}^H}{z^H}\right) + \varepsilon_t^H, \qquad 0 \leq \lambda^H < 1, \quad \varepsilon_t^H \sim iid\left(0, \sigma_H^2\right)
$$

The representative housing producer maximizes profits given (69), which yields the following optimality condition:

$$
Q_t^H = \left\{1 - \phi_1^H\left[\frac{I_t^H}{H_{t-1}} - (\delta_H - 1)z_t^H\right] - \phi_2^H\left(\frac{I_t^H}{H_{t-1}} - \frac{I_{t-1}^H}{H_{t-2}}\right)\right\}^{-1}
$$

where $Q_t^H \equiv \frac{P_t^H}{P_t}$ is the real house price. The housing stock evolves according to the following law of motion:

$$
H_t = (1 - \delta_H) H_{t-1} + IH_t
\tag{71}
$$

2.9   Exogenous variables

Foreign variables are exogenous to the Norwegian economy. The foreign economy could be modelled as a closed economy DSGE model, which is the approach taken in earlier versions of NEMO. Given that we condition on all relevant foreign variables in the forecasting procedure, we simply model foreign variables as univariate AR-processes. This assumption also apply to (domestic) public spending and oil investments. Hence,

foreign GDP, inflation and interest rates follow

$$\ln\left(\frac{T_t^*}{T^*}\right) = \lambda_u^* \ln\left(\frac{T_{t-1}^*}{T^*}\right) + \varepsilon_t^{u*}.$$

$$\ln\left(\frac{R_t^*}{R^*}\right) = \lambda_R^* \ln\left(\frac{R_{t-1}^*}{R^*}\right) + \varepsilon_t^{R*}.$$

$$\ln\left(\frac{P_t^*/P_{t-1}^*}{\pi^*}\right) = \lambda_P^* \ln\left(\frac{P_{t-1}^*/P_{t-2}^*}{\pi^*}\right) + \varepsilon_t^{P*}.$$

Finally, the government purchases final goods financed through a lump-sum tax, Ξ

$$P_t G_t = \Xi_t, \tag{72}$$

where G_t is real per capita government spending following an exogenous AR-process.

2.10 Equilibrium conditions

The model is closed by a set of market-clearing conditions, ensuring that demand equals supply. Supply of intermediates must equal demand. Furthermore, the supply of final goods must equal total demand as follows:

$$A_t = C_t + I_t + I_t^H + I_t^{OIL} + G_t \tag{73}$$

where

$$C_t = \varrho C_t^{im} + (1 - \varrho) C_t^{pa}$$

The intermediate good is used both at home and exported

$$T_t = Q_t + M_t^*.$$

Finally in equilibrium

$$H_t = \varrho H_t^{im} + (1 - \varrho) H_t^{pa} \tag{74}$$

2.11 Monetary policy

The government also controls the short-term interest rate, R_t. Interest rates are set with the objective to minimize the discounted sum of future (expected) losses. In the benchmark case, the period t loss is given by:

$$\mathcal{L}_t = \frac{1}{2}\left[\widehat{\pi}_t^2 + \lambda_y \widehat{y}_t^2\right]$$

In practice, policymakers might want to include elements in the loss function that take into account uncertainty and financial stability considerations.

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